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Journal of Electrostatics 40&41 (1997) 33–38

Journal of
ELECTROSTATICS

Absolute and convective instabilities in an electrified jet

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In this paper we discuss a linear stability analysis of a cylindrical jet flowing inside a coaxial cylindrical electrode in terms of absolute and convective instability concepts. This analysis show that a radial electric field applied on the jet surface may change the way an instability propagates in the flow.

1. INTRODUCTION

In general, liquid jets and droplets appear in industrial process where it is desired: to increase the surface/volume ratio (combustion, chemical reactors, ...) or to direct the liquid towards a target (painting, pesticide applications, ink jets printers,...). Different approaches to the breakup of the jets into drops at the different regimes observed when increasing the jet velocity (dripping, Rayleigh regime, wind regimes and atomization) have been done by mean of the stability analysis. The electrification of the jets has enabled a higher control of the breakup process as the electric forces change the stability of the jet.

The stability analysis is concerned with the development in space and time of infinitesimal perturbations in a given basic flow. As a function of the kind of perturbation imposed to the flow, two different problems, the temporal and the spatial ones, have been usually considered.

The temporal problem considers that the perturbation is applied to the flow in a certain region of the space at time equal zero. The development in Fourier series of this kind of perturbation gives components with real values of the wavenumber **k** and their variation in time is obtained with the complex frequencies $\omega(k) = \omega_r(k) + i\omega_i(k)$, roots of the dispersion equation $D(k,\omega) = 0$. The spatial problem considers that the perturbation is applied in a certain region and it follows a given function in time. A special case of this problem is the signaling problem where a perturbation is periodically forced at a specific location. The development in Fourier series of this kind of pertufbation gives components with real values of frequencies $k(\omega)$ and their variation in space is obtained with the complex wavenumbers $k(\omega) = k_r(\omega) + ik_i(\omega)$ deduced from the dispersion equation. Both analysis give the same information only when the Gaster's theorem conditions are verified [1] and in most cases we can not extrapolate results from one analysis to the other one.

A large part of previous research has been devoted to the temporal theory being L. Rayleigh one of the pioneers in this area [2]. This kind of analysis reveals to be quite important as it enables to predict the linearly stability of the flow (temporal growth rate positive), the interval of frequencies of the waves susceptible of being amplified in a signaling problem and the upper limit of the growth rate attainable in the system. For electrified jets this analysis considering the

0304-3886/97/$17.00
PII S0304-3886(97)00011-9

liquid viscosity and the surrounding atmosphere effect have recently been published [3-4] showing that for wavenumbers large enough a radial electric field destabilises the flow.

As some flows are extremely sensible to external noises, many controlled experiments have been undertaken to analyze the response or "receptivity of the flow" to different excitation frequencies. The spatial analysis has been used to describe these experiments analysing the downstream development of the perturbation as a set of spatially growing waves of various frequencies. We can conclude with this analysis which are the amplifying (evanescent) waves, that is to say those which amplitude increase (decrease) from the source of perturbation. In the case of electrified jets in [5] it is shown that the frequency domain of amplifying waves can be enlarged with a radial electric field applied on the surface of the jet.

A perturbation on the jet can be considered as a wavepacket, each component of the Fourier series being the simple waves that constitutes this packet. This packet may spread and increase with time in an unstable flow or it may dampen with time in a stable one.

As a function of the velocity of propagation of the wavepacket, for an unstable flow we can find two different situations. If the wavepacket is swept away from the source the instability is convective. For the second case, the absolutely unstable flow, the localised disturbances spread upstream and downstream from the source contaminating the whole flow. Some authors associate directly the absolute/convective instability character to subcritical/supercritical flows, where subcritical (supercritical) takes place when the velocity of the jet is smaller (larger) than the velocity of the wave packet. Then in the subcritical flow the perturbations are not swept away and encompass the source. In a mathematical context this analysis requires to consider in the dispersion equation simultaneously complex wavenumbers and complex frequencies.

The temporal and spatial analysis have been usually limited to the analysis of the local stability of the velocity profile in a typical streamwise station that is invariant with the axial co-ordinate. Strictly, a full analysis requires to consider the instability not only of the local velocity profile in a streamwise station as it does the local analysis, but of the entire flow field as it does the global one. Many recent theoretical efforts based on the temporal and absolute/convective analysis of the local velocity profile have been devoted to determine the relationship of local and global instability. In the case of electrified jets, the streamwise stations may differ one from the other as a result of i.e.: the initial velocity profile relaxation, the viscous action of the surrounding atmosphere, the limited time of charge relaxation, etc..

As a result, the theoretical framework considering absolute/convective and local/global stability concepts enable to better understand the qualitative nature of the dynamical behaviour of electrified jets. In this article we discuss the local stability analysis of a cylindrical jet flowing inside a coaxial cylindrical electrode, in terms of absolute and convective instability concepts. The more complex phenomena of global instability and the non linear effects in electrified jets like considered by Atten et al [6], though necessary to describe the whole phenomena, are not under the scope of this article.

2. MATHEMATICAL DESCRIPTION OF THE LOCAL INSTABILITY PROBLEM

Figure n°1 represents the non perturbed situation of the problem we study: It is an infinite liquid cylinder flowing at uniform velocity $\vec{U}_0$ in air at rest, and inside a coaxial electrode brought to a different electric potential. In our study we neglect the viscous effects, the mass transfer between phases and the gravitational and magnetic effects. Both fluids are considered incompressible and the liquid jet velocity is considered uniform in the cross section. We will accept that liquid relaxation time of charges is very short and so the jet surface is always an equipotential.

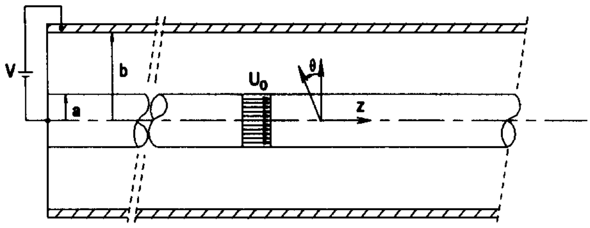

Figure 1 : Schema of the problem analysed

To analyse the convective or absolute character of an instability we use the Green function G. The impulse response, or Green function, of a flow is the instability-wave field generated by a Dirac delta function in space and time. The unstable Green function can evolve in two different ways: it can grow without bound at any point encompassing the source of perturbation (absolute instability) or it can grow and propagate away form the source (convective instability).

The expression of the Green function is obtained by the Fourier-Laplace integral:

$$G(z,\theta,t)=\sum_n \int_L \int_F e^{i(kz+n\theta-\omega t)} D^{-1}(\omega,n,k)\frac{dk}{2\pi}\frac{d\omega}{2\pi}$$

where $D(\omega,n,k)=0$ is the dispersion relation for complex frequencies ω and wavenumbers k. The Fourier contour is taken as the real axis of the complex plane k and the Laplace contour is a line parallel to the real axis of the complex plane ω above all singularities of $D^{-1}(\omega,n,k)$.

The dispersion relation of our problem obtained from a perturbation of the kind $\eta=\eta_0 e^{i(kz+n\theta-\omega t)}$ can be obtained in a similar way as described in [4,7]. From a co-ordinate frame fixed to laboratory, a linear analysis leads to the following dispersion equation:

$$\frac{\gamma_{i-j}}{a^2}k\left(1-n^2-(ak)^2\right)+\rho_1\Lambda(-\omega+kU_0)^2-\rho_2\,\alpha\,\omega^2-\frac{\varepsilon_0 E_n^2}{a}k(1+ak\varsigma)=0$$

where γ_{i-j} is the surface tension, a is the jet radius, ρ_1, ρ_2 are the jet and the gas densities respectively, ε_0 is the vacuum dielectric constant, E_n is the normal electric field in the non perturbed situation, and

$$\Lambda=\frac{I_n(ka)}{I_n'(ka)},\ \alpha=\frac{\Lambda}{(1-\lambda)}+\frac{K_n(ka)}{K_n'(ka)\left(1-\frac{1}{\lambda}\right)},\ \varsigma=\frac{1}{\Lambda(1-\beta)}+\frac{K_n'(ka)}{K_n(ka)\left(1-\frac{1}{\beta}\right)},$$

$$\lambda=\frac{I_n'(kb)\,K_n'(ka)}{K_n'(kb)\,I_n'(ka)},\ \beta=\frac{I_n(kb)\,K_n(ka)}{K_n(kb)\,I_n(ka)}$$

where In, Kn are the modified Bessel functions of first and second kind, and ' indicates the derivative with respect to the argument. When the electrode radius b>>a, we can use $\alpha = \frac{K_n(ka)}{K_n'(ka)}, \varsigma = \frac{1}{\alpha}$. Considering $\varpi = \omega \frac{a}{U_0}$, $\kappa = ak$ and the development of the dispersion equation in its real and imaginary part we obtain :

$$A\kappa_r^3 + B\kappa_r^2 + C\kappa_r + D\kappa_r\kappa_i^2 + E\kappa_r\kappa_i + F\kappa_i^2 + G\kappa_i + H = 0$$
$$I\kappa_i^3 + J\kappa_i^2 + K\kappa_i + L\kappa_i\kappa_r^2 + M\kappa_i\kappa_r + N\kappa_r^2 + O\kappa_r + P = 0$$

$$\begin{array}{ll}
A = -E_u & I = E_u \\
B = \Lambda_r - E_{ue}\varsigma_r & J = -\Lambda_i + E_{ue}\varsigma_i \\
C = E_u(1-n^2) - E_{ue} + 2(\Lambda_i\varpi_i - \Lambda_r\varpi_r) & K = C \\
D = 3E_u & L = -3E_u \\
E = -2\Lambda_i + 2E_{ue}\varsigma_i & M = 2(\Lambda_r - E_{ue}\varsigma_r) \\
F = -\Lambda_r + E_{ue}\varsigma_r & N = \Lambda_i - E_{ue}\varsigma_i \\
G = 2(\Lambda_i\varpi_r + \Lambda_r\varpi_i) & O = -2(\Lambda_r\varpi_i + \Lambda_i\varpi_r) \\
H = (\varpi_i^2 - \varpi_r^2)(-\Lambda_r + r_d\alpha_r) - 2\varpi_r\varpi_i(\Lambda_i - r_d\alpha_i) & \\
P = (\varpi_i^2 - \varpi_r^2)(-\Lambda_i + r_d\alpha_i) - 2\varpi_r\varpi_i(-\Lambda_r + r_d\alpha_r) &
\end{array}$$

and the subscript r indicates real part and i complex part. We can see then, that the parameters of the problem are reduced to the density ratio $r_d = \frac{\rho_2}{\rho_1}$, the Euler number $E_u = \frac{\gamma_{i-j}}{\rho_1 U_0^2 a}$, and the electrical Euler number $E_{ue} = \frac{\varepsilon_0 E_n^2}{\rho_1 U_0^2}$.

The criteria established in [8-9] to determine if the flow is absolute unstable requires to deform the Laplace contour towards the lower half ω plane and obtain its image in the k plane. This process of consecutive contour deformation enables to detect the "pinch" points k_0 of the mapping. They are the saddle points of the function k(ω). At these points the group velocity $d\omega/dk$ is null. If the corresponding branch point $\omega_0 = \omega(k_0)$ in the ω plane has a positive imaginary component it will dominate the time asymptotic response and is the case of absolute instability. The point ω_0 is called commonly the absolute frequency and $\omega_{i_{abs}} = \omega_i(k_0)$ is the absolute growth rate. On the other hand, if the Laplace contour can be deformed to the real ω axis without appearing any pinch point then the instability is convective.

2.1 Numerical solution

Our function κ(ϖ), has three different solution for a same value of ϖ and n, so we can detect three branches in the κ plane for different n. To detect the three branches we use a grid search technique similar to the one described by Tam and Hu [10]. This technique consists to calculate at the crossing points of a grid placed in the region of interest of the ϖ plane, the values of $Re(D(\varpi,\kappa))$ and $Im(D(\varpi,\kappa))$. This enables to estimate the two families of curves $Re(D(\varpi,\kappa))\approx 0$ and $Im(D(\varpi,\kappa))\approx 0$. The intersections of this families are considered as the initial estimation of the zeros of $D(\varpi,\kappa)$, which are obtained with a subroutine for solving systems of non-linear equations based on the Powell method [11]. The Figure 2 shows an example of a pinch point in an electrified jet considering the first mode (n=0) and a given set of parameters.

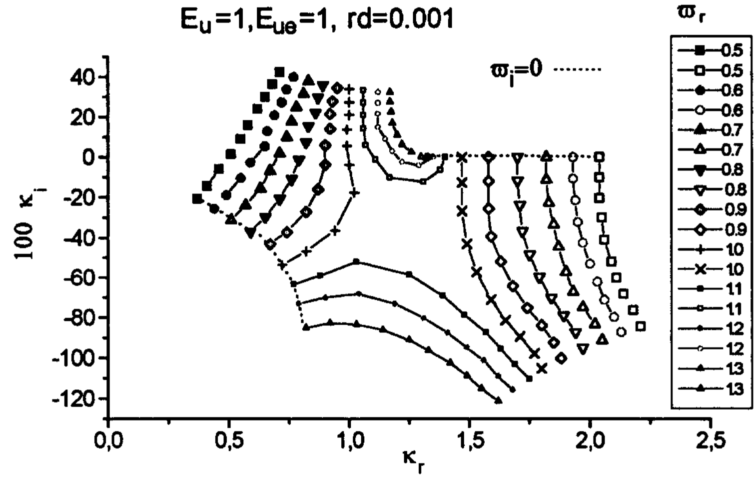

Figure 2: Typical Pinch Point

The figure 3 shows the dependency of the absolute growth rate $\varpi_{i_{abs}}$ on the value of E_{ue} for a typical case. The regions where the instability of the flow are absolute (absolute growth rate positive) or convective (absolute growth rate negative) are indicated.

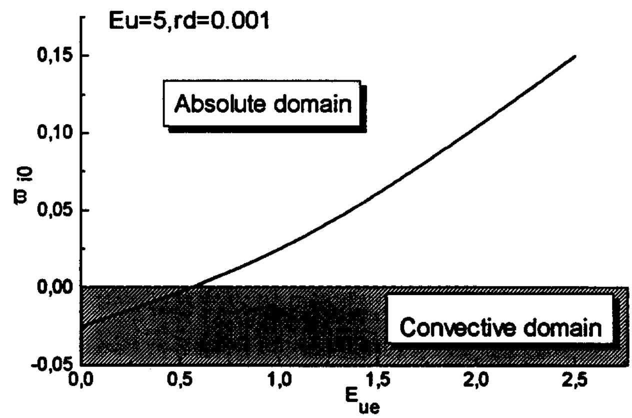

Figure 3: Absolute growth rate dependency with electric field

3. DISCUSSION

The role of a parameter as far as convective or absolute instability concepts is established observing if an increase of the value of the parameter tends to increase or decrease the value of absolute growth rate. As the velocity of propagation of the wavepacket is null when the instability is absolute, the tendency of the absolute growth rate towards the absolute domain by increasing the value of a parameter can be understood as a deceleration of the velocity of the wave packet and so a tendency towards the subcritical flow.

Other researchers have demonstrated which is the role of the parameters E_u [12] and of r_d [13], concerning the absolute or convective character of non electrified jets. In [12] it is shown that for a given set of parameters an increase of the jet velocity (or of $We=1/E_u$) tends to diminish the absolute growth rate. So when We increases the velocity of the wave propagation increases, that is to say the flow "tends to be more convective". The transition of the dripping regime to the fully established jet observed when we increase the exit velocity of a liquid from an orifice may be explained with this kind of analysis. On the other hand, an increase of the density ratios tends to increase the absolute growth rate [13], the velocity of the wave propagation decreases, and so the flow "tends to be more absolute".

From the analysis of figures 3, we can see this last behaviour is also obtained when electrifying a high conducting jet. In this case, by increasing the intensity of the electric field of the basic state (or of E_{ue}) we may change the instability of the flow from convective to absolute. So the electric field decelerates the velocity of the wave propagation and it may change the flow from supercritical to subcritical.

4. CONCLUSION

In this paper we have clarified the role of the electric field concerning the concepts of local absolute and convective stability. We have analysed the case when the liquid surface is an equipotential and demonstrated the decelerating role of the electric field for the velocity of the wave propagation. As a further work we plan to do more efforts to achieve a global analysis.

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