# Global Instabilities in an electrified jet

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**Abstract.** This article describes the influence of an electric field on the stability of an axially accelerated liquid jet. We present the fundamental data to determine the global stability for any case and particularly study the case when the jet is accelerated mainly by gravitational forces. This research indicates that dangerous situations of global stability loss occur for the cases of strong electric field close to the jet exit and low velocity jets.

## 1. Introduction

In the stability analysis of electrified jets as a function of the type of perturbation imposed to the flow, two different problems, the temporal and spatial one, have usually been considered. The temporal problem considers that the perturbation is applied to the flow in a certain region of the space at time equal zero, while the spatial problem considers that the perturbation is applied in a certain region and follows a given function in time. These kind of analyses have usually been restricted to the local stability of the velocity profile in a typical streamwise station that was invariant in the direction of the wave propagation. However, when the basic flow differs significantly from one streamwise station to the other, the stability analysis requires to consider the stability not only of the local velocity profile in a typical station (local stability analysis), but of the entire flow field (global stability analysis). In the particular case of electrified jets results obtained from global analysis are of great interest. The streamwise stations may differ from each other as a result, among other phenomena, of gravity acceleration, of initial velocity profile relaxation, of viscous action of the surrounding atmosphere and of the limited time of charge relaxation that leads to a non equipotential jet surface.

Previous research dealing with electrified jets has tried to gain some insight of this kind of problem, and the effect of the acceleration of the jet due to gravitational forces on the growth rate of perturbations imposed to the flow has received special attention [1] . However, at that time the theory of global instability was not fully developed and concepts of locally and globally stable flows were not applied. Recently the global response of a gravitational jet flow acting either as an amplifier that selectively amplifies extrinsic noise (global stability) or as an oscillator of self-excited time-growing oscillations at any fixed location (global instability) has been established [2-3]. It is the objective of this article to identify in electrified jets the flow and electric field configuration that leads to either behaviour.

## 2. Description

We analyze the case of a downwards jet issuing from a circular nozzle coaxial with a cylindrical electrode. Hypothesis considered are: the surface of the liquid jet is an electrical equipotential; the magnetic effects can be disregarded; the jet is inviscid, incompressible and isothermal and there is no mass transfer between the jet and the surrounding atmosphere.

### *2.1. Local Analysis : Absolute and Convective flows*

The link between the local stability properties and the global behaviour of the flow is still controversial. Nevertheless, most observations support the idea that the formation of local absolute instability pockets is a necessary condition for the appearance of a global instability. The absolute or convective nature of an instability is related to the asymptotic impulse response for time t$\rightarrow \infty$ . Convectively unstable flows give rise to wave packets that move away from the source and ultimately leave the medium in its undisturbed state. Absolutely unstable flows, by contrast, are gradually contaminated everywhere by a point-source input. In what follows we undertake this analysis by using the dispersion equation obtained in [4] considering negligible the effect of the sourrounding atmosphere and also that the electrode radius is much larger than the jet radius. This equation $\mathrm{D}(k, \omega, n, We, Eue) = 0$ establishes a relationship between the complex wavenumber $k$, the complex frequency $\omega$, the mode number $n$, the Weber number $We$ (the ratio between inertial and surface tension forces) and the Electric Euler number $Eue$ (the ratio between the electric forces and inertial forces ). Using this equation, and analysing the spatial branches in the k-plane [5] it is possible to obtain the absolute growth rate that characterizes the temporal evolution of the wave number of zero group velocity at a fixed station in the limit $t \rightarrow \infty$. A positive absolute growth rate is therefore associated with an absolute instability, and a negative or zero value with a convective one. Figure 1 represent this absolute growth rate (non dimensionalized with jet radius and velocity) as a function of mode number for different $Eue = \frac{\epsilon_0 E_n^2}{\rho U^2}$ (where $\epsilon_0$ is the dielectric constant in vaccum, $E_n$ the electric field at the jet surface, $\rho$ the liquid density and $U$ the mean jet velocity). It can be observed in this figure that, as the first mode is the one which has the highest absolute growth rate, in general we can limit the analysis to this mode to observe the absolute/convective unstable behaviour of the electrified jet. Figure 2 shows the absolute growth rate as a function of the ratio of electric forces with surface tension forces ($Rae = Eue * We$) for different Weber number ($We = \frac{\rho U^2 a}{\gamma}$, with $a$ the jet radius and $\gamma$ the surface tension). This figure shows that for low Electric Euler number the absolute growth rate is null and also how by increasing the electric field it is possible to obtain positive values of the absolute growth rates. In Figure 3 we represent the critical Electric Euler Number $cr.Eue$ as a function of Weber number (critical Euler number is the one that separates absolute from convective behaviour). This last figure indicates that higher Weber number are associated with lower $cr.Eue$ ratios.

### 2.2. Global modes in weakly inhomogeneous flow

Under the umbrella of the hypotheses cited above the shape of the jet for a given nozzle diameter and flow rate will depend on the relative magnitude of gravitational, surface tension and electrical forces. Analytical expressions are difficult to obtain as one of the boundary conditions is part of the solution, and so numerical methods seem the most adequate alternative to obtain refined solutions. However in some limit scenarios simplified analytical solutions can be easily obtained. Results that are given below correspond to the extreme case occurring when the acceleration of the jet is determined mainly by gravitational forces. To obtain the jet shape we use the expressions deduced in [5] where it is determined that the ratio $\tau = a/a_0$ can be obtained from:

$$\left(\frac{1}{\tau^4} - 1\right) + \frac{2}{We_0}\left(\frac{1}{\tau} - 1\right) = 2gz\left(\frac{\pi a_0}{q}\right)^2$$

with $We_0$ and $a_0$ the Weber number and the jet radius at $z = 0$, $g$ gravity acceleration, $z$ axial coordinate, and $q$ the flow rate. Links between local and global analysis are obtainable when on the scale of the instability wavelength the base flows develops slowly in the streamwise direction [2]. This condition is satisfied in this case if $ga_0^3/q^2 \ll 1$. Using the above expression and considering an electric field corresponding to corona inception we show in figure 4 the non-dimensional growth rate as a function of the axial position. This figure indicates that pockets of absolute instability may appear and so global stability be lost if the electric field strongly stresses the jet surface close to the jet exit.

In some cases where gravitational forces are not predominant, it may be of interest to have some rough estimations of the limits of global stability of electrified jets without undertaking numerical analysis to obtain the velocity and electric field variation with the axial coordinate. Looking at figure 3 we may see in this figure that it seems reasonable for many cases of interest ($We$ close to or lower than 100) to accept as a limiting value for global stability the ratio $Eue \cong 0.35$. By doing that and considering at the jet surface the maximum ionization field in air, it can be stated that no global instability should appear for values of $Uo \leq \sqrt{227.7/\rho}$ with the liquid density $\rho$ in SI units. In contrast, if it is desired to design an experiment to observe global instabilities by stressing a water jet with an electric field, by using the above criteria the jet velocity $U_0$ should at least be lesser than a value of 0.47m/s but it should be pointed out that the nozzle radius should assure an Weber number large enough to avoid dripping regime ($We \succeq 3.1$).

## 3. Conclusions

This research work establishes the set of Weber and electric Euler number that assures dampening of the global mode for the case of highly conducting inviscid liquids. We have considered a simple case of predominant gravity acceleration of the jet. For this case pockets of absolute instability that could lead to a global instability may appear only for strong electric field close to the jet exit. This research also indicates that in general, flows with low velocity jets stressed with strong electric fields are situations of special attention where global stability must be analysed carefully.

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## References


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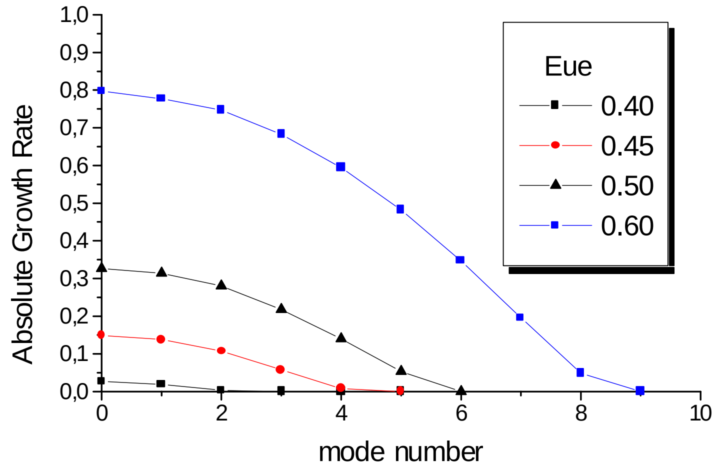


Figure 1: Non dimensional absolute growth rate as a function of mode number n for different Electric Euler Number

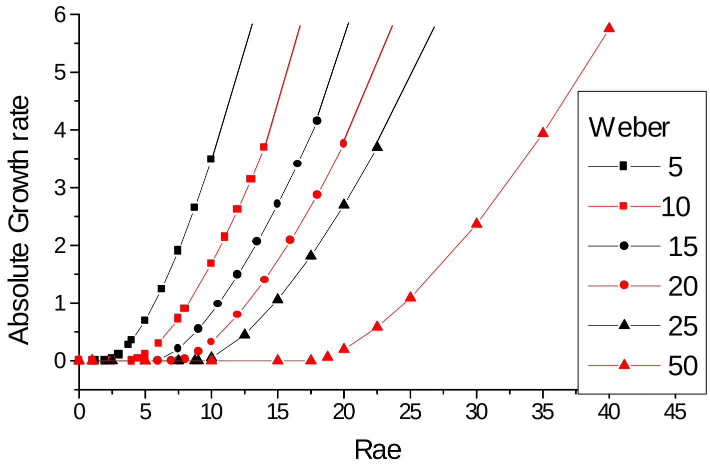


Figure 2: Non dimensional absolute growth rate as a function of Rae for different Weber number

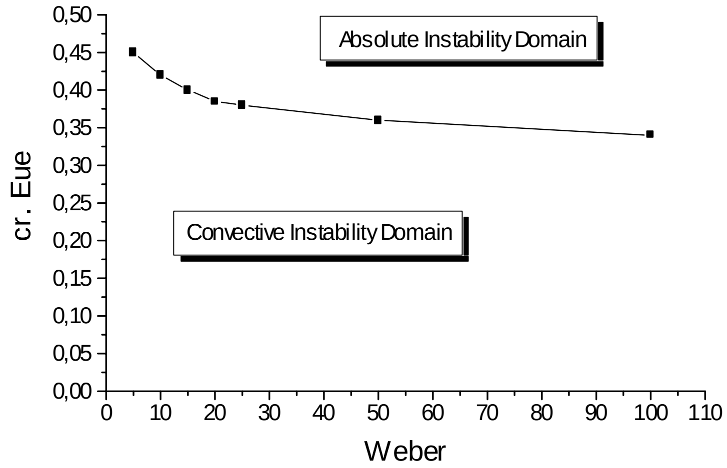


Figure 3: Critical Electrical Euler Number as a function of Weber number

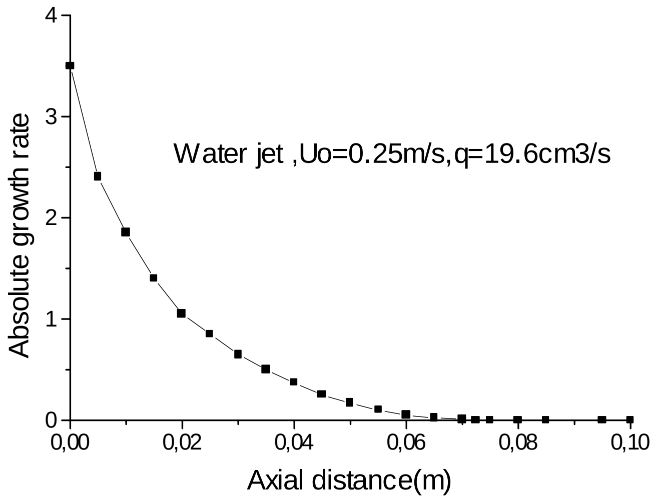


Figure 4: Nondimensional absolute growth rate vs axial distance for a gravitational jet.