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Theoretical influence of the pressure of the surrounding atmosphere on the stability of high velocity jets

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In this article, we study the theorical influence of the pressure of the surrounding atmosphere on the stability of high velocity jets submitted to an electric field. The analysis of the stability of the electrified jet is linear, temporal and in normal modes. It consists in an expansion of a perturbation (in our case in Fourier integral) and in studying the stability of each single component of the expansion.

In the first part of this article, we briefly explain how to determine the equation of the impulsion conservation at the interface after perturbation. The resolution of this equation called dispersion equation gives the values of the wave numbers k and the growth rates w for which the jet is unstable.

In a second part, we show the influence of the pressure, the electric field, the velocity and of the number of mode on the stability of the jet.

1. THEORY

1.1. Description of the non perturbed system

We will consider an infinite liquid cylinder of density ρ_1 and with a radius a. It flows away with a uniform velocity $\vec{U}_0$ into a cylindric coaxial electrode whose radius is b. V is the value of the electrical potential at b. The liquid is supposed to be incompressible and inviscid. A gazeous atmosphere with density ρ_2, supposed incompressible and inviscid, is situated between the liquid cylinder and the electrode.

Moreover, we will assume that :

- there is no mass transfer between the two phases,
- the liquid velocity is uniform in the jet section,
- the effects caused by the gravitational and magnetic fields are negligible.

We will work in a cylindric coaxial frame of reference $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$ which moves with the jet velocity $\vec{U}_0$.

1.2. Linear study of the stability

Artana [1-2] wrote the general equations (corresponding to the mass conservation and the impulsion conservation) of the phenomenon in each phase and at the interface as well. Let us sum up his work.

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At a certain time t, we impose an infinitesimal perturbation $\eta = \eta_0 \exp[i(kz + n\theta) + wt]$ at the surface of the jet, and we study its evolution in the time. The writing of the equation of the impulsion conservation at the interface after perturbation requires the expressions of the velocity, electric and pressure fields in the two phases after perturbation. These expressions come from the general equations.

Referring to [1-2], the velocity in the liquid phase is : $\vec{U}_1 = \dfrac{w}{kI_n'(ka)}\vec{\nabla}(I_n(kr)\eta)$ and the velocity in the gazeous phase is :

$$\vec{U}_2 = \vec{U}_0 + \frac{w - iU_0 k}{k}\vec{\nabla}\left(\left(\frac{I_n(kr)}{I_n'(ka)(1-\lambda)} + \frac{K_n(kr)}{K_n'(ka)(1-1/\lambda)}\right)\eta\right) \qquad \text{with } \lambda = \frac{I_n'(kb)K_n'(ka)}{K_n'(kb)I_n'(ka)}$$

where I_n is the modified Bessel function of the first kind and K_n is the modified Bessel function of the second kind.

For an equipotential surface the electric field is, in the liquid phase : $\vec{E}_{1e} = \vec{0}$ and in the gazeous phase : $\vec{E}_{2e} = \dfrac{V}{\ln(b/a)}\vec{\nabla}\left(\ln(r/a) - \left(\dfrac{I_n(kr)}{aI_n(ka)(1-\beta)} + \dfrac{K_n(kr)}{aK_n(ka)(1-\frac{1}{\beta})}\right)\eta\right)$, where $\beta = \dfrac{K_n(ka)I_n(kb)}{I_n(ka)K_n(kb)}$

For a non equipotential surface the electric field is,

in the liquid phase : $\vec{E}_{1ne} = \vec{\nabla}\left(\dfrac{V I_n(kr)}{a\ln(b/a)I_n(ka)\left(1 - \dfrac{\varepsilon_1}{\varepsilon_2}\dfrac{I_n'(ka)K_n(ka)}{I_n(ka)K_n'(ka)}\right)}\eta\right)$

and in the gazeous phase : $\vec{E}_{2ne} = -\dfrac{V}{\ln(b/a)}\vec{\nabla}\left(\ln(r/a) - \dfrac{K_n(kr)}{aK_n(ka)\left(\dfrac{\varepsilon_2}{\varepsilon_1}\dfrac{K_n'(ka)I_n(ka)}{K_n(ka)I_n'(ka)} - 1\right)}\eta\right)$

The equation of the impulsion conservation, in each phase with the last velocity and electric fields gives the pressure p_1 and p_2 in each phase. So, we can write the expression of the impulsion conservation at the interface which is :

$$\frac{\gamma_{i-j}}{a^2}(1 - n^2 - (ak)^2) - \rho_1\frac{I_n(ka)}{I_n'(ka)}\frac{w^2}{k} + \rho_2\frac{(w - iU_0 k)^2}{k}\alpha - \varepsilon_o\frac{V^2}{a^2\ln^2(b/a)}\left(\frac{1}{a} + k\xi\right) = 0 \qquad (1)$$

where $\alpha = \dfrac{I_n(ka)}{I_n'(ka)(1-\lambda)} + \dfrac{K_n(ka)}{K_n'(ka)\left(1 - \dfrac{1}{\lambda}\right)}$

In formula (1) $\xi = \xi_e$ in the equipotential case and $\xi = \xi_{ne}$ in the non equipotential case, where ξ_e and ξ_{ne} are :

$$\xi_e = \frac{I_n'(ka)}{I_n(ka)(1-\beta)} + \frac{K_n'(ka)}{K_n(ka)\left(1-\frac{1}{\beta}\right)} \quad \text{and} \quad \xi_{ne} = -\frac{1}{\frac{K_n(ka)}{K_n'(ka)} - \frac{\varepsilon_2}{\varepsilon_1}\frac{I_n(ka)}{I_n'(ka)}}$$

In a temporal analysis, the growth rate w is complex : $w(k)=w_r(k)+iw_i(k)$. The equation of dispersion (1) gives us two equations, one for the real part and another one for the imaginary part which are :

$$w_r^2 = \frac{k}{a^2(1-\chi)\rho_2\alpha}\left[-\gamma_{i-j}(1-n^2-(ak)^2)+\varepsilon_0\frac{V^2}{a\ln^2(b/a)}(1+ak\xi)\right]-w_i^2\chi \tag{2}$$

$$w_i = \frac{U_o k}{1-\chi} \qquad \text{where } \chi = \frac{\rho_1 I_n(ka)}{\rho_2 I_n'(ka)\alpha}$$

The real part of w is the expression of the growth rate of the perturbation. If it is positive, the system is unstable and the amplitude of the perturbation grows with time. If it is negative, the system is then stable and the perturbation vanishes. We present now the intervals within which the wave number of the jet is unstable. More details can be found in [1].

2. PRESENTATION OF RESULTS AND DISCUSSION

We give some results that come from the resolution of equation (2). We take $a=100.10^{-6}$ m for the radius of the jet, $b=500.10^{-6}$ m for the radius of the nozzle and $\gamma_{i-j}=72{,}2.10^{-3}$ N/m for the surface tension water. The density ρ_2 which depends on the pressure of the surrounding atmosphere of the jet is given by : $\rho_2=p/RT$ where p is the pressure of the atmosphere, T the temperature (T=25°C) and R the constant of the gas ($R=287\ m^2s^{2}K^{1}$).

From a temporal analysis on high velocity jets, but for non electrified jets, different authors [3-4-5] established the expressions of the average diameter $\bar{\delta}$ of the droplets ejected in the atmosphere and the intact length L_0. The average diameter is $\bar{\delta} = 2\pi A / k_{max}$ where A is a constant and the intact length is $L_0 \approx C / w_{r\,max}$ where C is another constant. The resolution of the dispersion equation gives the variation of the growth rate w_r as a function of k, which gives in its turn the position of the maximum and therefore $\bar{\delta}$ and L_0.

2.1. Influence of the pressure

The next three graphs show the variation of the real frequency w_r as a function of ak, for different pressures (p=10 to 50 atm). The coordinates of the maximum are : $w_{r\,max}$ and ak_{max}.

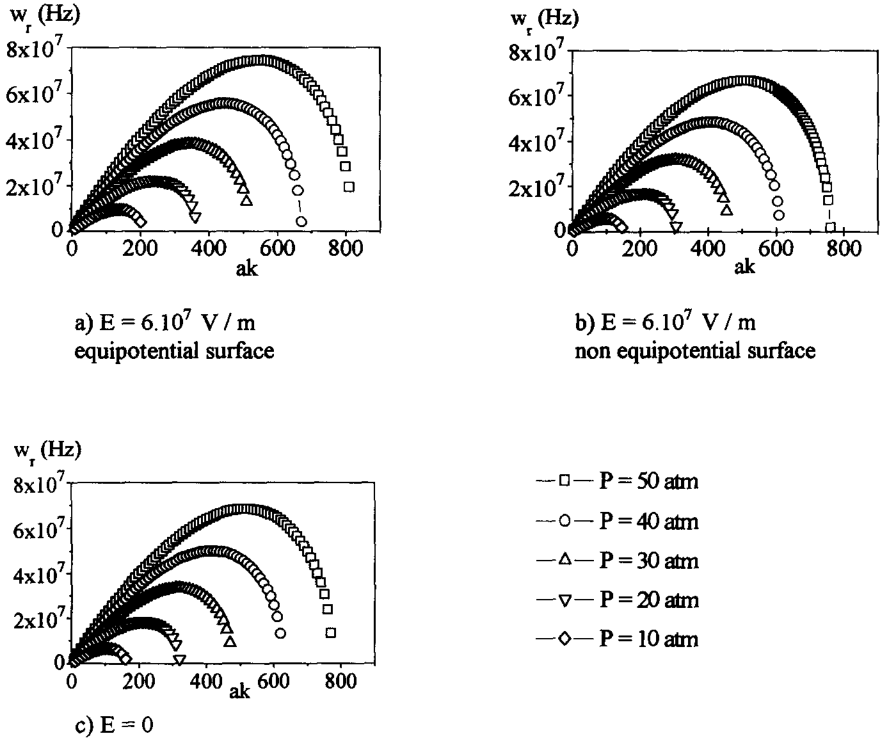

Figure 1 : Influence of the pressure, n=0, U_0=100 m / s

These three graphs show that an increase of the pressure induces an increase in w_r whatever the case. We can also notice that the more the pressure increases the higher the value of ak_{max}. An increase in the pressure enlarges the interval of ak for which the jet is unstable.

Therefore, the pressure is a destabilizing parameter - for a fixed k an increase in the pressure induces an increase in the growth rate w_r thus an amplification of the amplitude of the perturbation. This finally proves that the higher the pressure, the smaller the size of the droplets and the nearest to the nozzle they will break off. These results are valid for a non electrified jet and for an electrified one (equipotential surface and non equipotential surface).

With the help of graphs 1(b) and 1(c) we can see that in the non equipotential case the values of w_r are lightly smaller than those of the non electrified jet. On the other hand, graphs 1(a) and 1(c) show that we obtain values for w_r higher for the electrified jets (equipotential surface) than for the non electrified ones and also show that the values of the maxima ak_{max} are always more important. We can conclude that, in the equipotential case, the electric field diminishes the size of the droplets and the intact length L_0 (the droplets break off nearer to the nozzle).

2.2. Influence of the electric field

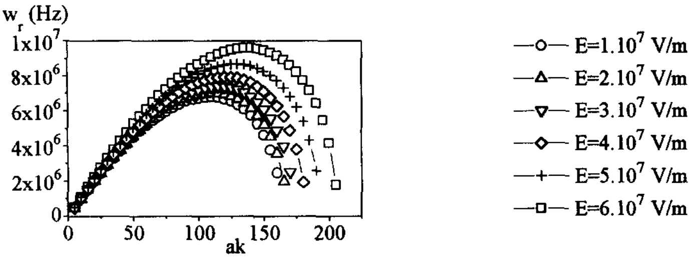

Figure 2 : Influence of the electric field. Equipotential surface, n = 0, U_0=100 m/s, p = 10 atm

This figure shows that the higher the field the more w_r and ak_{max} increase. Therefore, the higher the field, the smaller the size of the droplets and the nearest to the nozzle they will break off. In the equipotential case, the electric field is therefore a destabilizing parameter. We have noticed this phenomenon for different other pressures.

2.3. Influence of the velocity

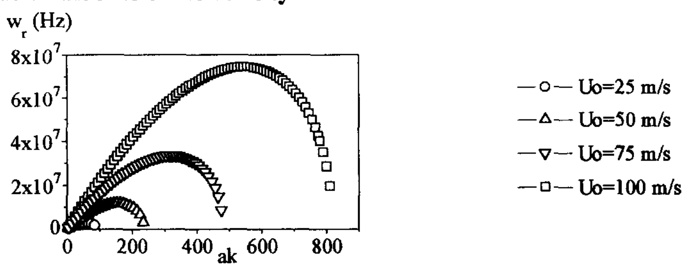

Figure 3 : Influence of the velocity. Equipotential surface, n=0, p=50 atm, E=6.10^7 V/m

Figure 3 shows that the velocity has the same influence as the electric field. The velocity is a destabilizing parameter, which has been noticed for different other pressures.

2.4. Influence of the number of mode

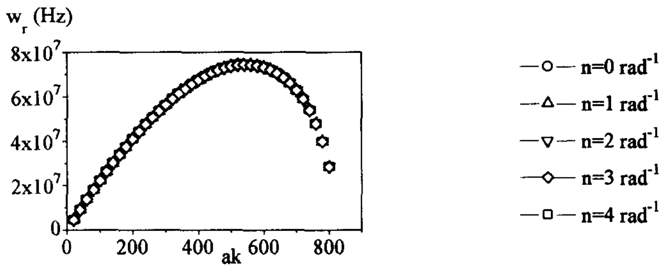

Figure 4 : Influence of the number of mode. Equipotential surface, p=50 atm, U_0=100 m/s, E=6.10^7 V/m

We did the same calculation as the one reported in figure 1 a) with n=1 instead of n=0. We found the same results for different pressures. On figure 4 we can see that w_r remains the same when the mode number changes (n=0 to n=4). The mode number has no influence on the stability of the jet for these pressures.

3. PERSPECTIVES

At the moment, we are studying the nonlinear stability of an electrified jet. We use a straightforward expansion for the perturbation, the potential velocity and the electric potential as Lafrance did [6].

$$\eta(\theta,z,t)=\sum_{n=1}^{+\infty}\eta_0^n\eta_n(\theta,z,t);\quad \varphi(r,\theta,z,t)=\sum_{n=0}^{+\infty}\eta_0^n\varphi_n(r,\theta,z,t);\quad \phi(r,\theta,z,t)=\sum_{n=0}^{+\infty}\eta_0^n\phi_n(r,\theta,z,t)$$

in which η_0 is the smallness parameter. Then we obtain the velocity and electric fields after perturbation and at the second order. The equilibrium at the interface gives the dispersion equation which now contains a new term in η_0^2. We are working on this further term and on the modifications that it induces on the stability of the jet. We are also developping the multiple time scale method used by Nayfeh [7], which will give us more information on the stability.

4. CONCLUSION

The equation of the impulsion conservation at the interface gives us the possibility of determining the values of the wave numbers k and the frequencies w for which the amplitudes of the perturbations increase, that is to say for which the jet is unstable. The study of the real part of this function w(k) gives us information on the stability of the spray.

We have analyzed the influence of different parameters on the stability of the high velocity jet. The pressure, the electric field in the case of an equipotential surface and the velocity all have a destabilizing influence on the electrified jet. However, the mode number for our pressures has no influence on the stability of the jet.

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