Flow electrification in power transformers: electrical modeling

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Abstract: A charge zone named electrical double layer exists at a solid-liquid interface. The liquid flow induces a phenomenon called Flow Electrification: it generates a streaming current (caused by the convection of charges present in the liquid) and a potential rise in the solid (if this one is insulated from the ground). These potentials may reach values high enough to produce electrical discharges and provoke accidents. Although this phenomenon has been identified a long time ago, its physical description rests unknown (production and displacement of charges, equilibrium, etc.). As part of the research program of Electricité de France, a prototype sensor dedicated to flow electrification characterization at the interface between pressboard and oil has been developed with the University of Poitiers. In this paper we propose to use an electrical analogy to model these processes in the prototype. This electrical model is based on the analysis of experiences carried out with the sensor.

Introduction

As soon as a liquid gets in contact with a solid, the solid-liquid couple that was initially neutral becomes polarized under physicochemical reactions occurring at the interface. Such phenomenon leads to a space charge in the liquid (which can afterwards relax in contact with a grounded metallic recipient), and to a space charge in the solid, which can accumulate according to leakage paths [1].

Charge convection in the liquid creates a current called Streaming Current, and leads to a continuous charge "separation" process at the interface. When the solid is insulated from the ground, leakage impedances limit the accumulation of these charges at the wall.

In our case, the liquid used is transformer oil, and the solid is a rectangular pressboard duct, which is inserted in a PTFE frame.

Charge leakage takes place towards two stainless steel couplings placed at both extremities of the duct, and insulated from the rest of the loop by PTFE flange couplings.

Two plane electrodes are placed facing the external surfaces of the pressboard duct, beyond 2 mm of PTFE, to measure a mainly capacitive current (I_{ACC}) related to the charge trapped inside the pressboard (accumulation charges).

A complete description of the facility used to obtain the experimental data, as well as an analysis of static and dynamic equilibriums can be found in [1-2].

In this paper, we model the processes taking place in the facility by an electrical analogy. In the near future this will allow us to study the influence of different parameters over phenomenon (oil properties, temperature, mean flow velocity, etc).

Electrical modeling

Equivalent electrical schema

Figure 2 shows the electrical scheme used.

In this model, the charge separation mechanism is associated with current generators whose sum is at any time equal to the sum of the accumulation current and both of the leakage currents.

The π type array of resistances and capacitors represents the solid (pressboard-PTFE) and the interface.

We will refer to the center of each "cell" as node 1, node 2, node n.
Calculation of current generators

We will consider a rectangular duct of length \( L \), whose thickness \( 2a \) is negligible relative to its width \( D \) (figure 3). Thus, the assumption of two parallel plates is possible.

![Figure 3: Rectangular duct and leakage surface.](image)

We make the hypothesis that the charge separation process is related to a redox chemical reaction at the interface [3-4]. We also consider that our problem may be included among those of weak space charge density. This means that the liquid's electrical conductivity does not show great changes. Although this is usually not verified, Touchard has showed that the error made when the weak space charge approximation is used is often a few percent [5]. This is in the same order of magnitude of experiments accuracy. As a result, wall current density may be expressed as:

\[
\rho_{\text{w}}(z,t) = K \left[ \rho_{\text{sd}} - \rho_{\text{w}}(z,t) \right]
\]  

(1)

where \( K \) is a coefficient constant for a given physico-chemical reaction, \( \rho_{\text{sd}} \) is the space charge density near the interface and \( \rho_{\text{w}} \) is the space charge density near the interface for a fully developed double layer.

We assume from now on that the time needed for the development of the diffuse layer is much shorter than the time needed for the development of space charge at the interface and also shorter than the residence time of a particle in the duct due to convection. With these hypotheses, the space charge density inside the liquid is given by [6]:

\[
\rho(x,z,t) = \rho_{\text{sd}}(z,t) \left( 1 + a \frac{\cosh(x/\delta_0)}{\cosh(a/\delta_0)} \right)
\]  

(2)

\( \delta_0 \) being the diffuse layer thickness (also called the Debye length) which can be expressed as:

\[
\delta_0 = \sqrt{\frac{\varepsilon D_0}{\sigma_0}}
\]  

(3)

where \( \varepsilon \) is the liquid's dielectric constant, \( \sigma_0 \) its bulk conductivity and \( D_0 \) a mean diffusion coefficient.

The velocity profile for the laminar oil flow is:

\[
U(x) = \frac{3}{2} U_m \left( 1 - x^2 \right)
\]  

(4)

with \( U_m \) the mean flow velocity.

The streaming current due to the convection of the charges in the liquid is:

\[
I_{\text{stream}}(z,t) = \int_{-a}^{a} \rho(x,z,t) U(x) \, dx \, dy
\]  

(5)

The derivative of the charge conservation's integral equation, with respect to the axial coordinate \( z \) leads to:

\[
\frac{1}{D} \frac{\partial I_{\text{stream}}(z,t)}{\partial z} - 2i_{\text{w}}(z,t) = -\frac{\partial \rho(x,z,t)}{\partial t} \, dx
\]  

(6)

After replacing \( \rho_{\text{w}} \) and \( \rho \) according to (1) and (2), and integrating, (6) becomes:

\[
\beta \frac{\partial \rho_{\text{sd}}(z,t)}{\partial z} + \alpha \frac{\partial \rho_{\text{sd}}(z,t)}{\partial t} = -\alpha \beta \rho_{\text{sd}}(z,t) + \alpha \beta \rho_{\text{ad}}
\]  

(7)

where \( \alpha \) and \( \beta \) are constants given by:

\[
\beta = \frac{K}{\delta_0 \tanh \left( \frac{a}{\delta_0} \right)}
\]  

(8)

\[
\alpha = \frac{K}{\delta_0 \tanh \left( \frac{a}{\delta_0} \right)} U_m \left( 1 - \delta_0 \tanh \left( \frac{a}{\delta_0} \right) \right)
\]  

(9)

The solution of (7) has the following form:

\[
\rho_{\text{sd}}(z,t) = \rho_{\text{ad}} + \exp(-\alpha \varepsilon \Phi(\alpha \varepsilon - \beta t))
\]  

(10)

in which \( \Phi(u) \) is an arbitrary function.

The boundary conditions for the space charge density near the interface are (11), (12) and (13):

\[
\lim_{\tau \to +\infty} \rho_{\text{sd}}(z,t) = \rho_{\text{ad}} \left[ 1 - \exp(-\alpha \varepsilon \Phi) \right]
\]  

(11)

\( \rho_{\text{sd}}(z,t) \) is the solution of (7) when the time derivative is null.

\[
\rho_{\text{sd}}(z,0) = \text{const} = \rho_{\text{ad}}
\]  

(12)

This condition is due to the hypothesis that the flow sweeps a static double layer, which was in equilibrium.
\[
\lim_{z \to \infty} \rho_w(z,t) = \rho_{wd}
\]  

(13)

According to (10) and these boundary conditions, the space charge density near the interface can be expressed by:

\[
\rho_n(z,t) = \rho_{wd}[1 - \exp(-\alpha z)[1 - \exp(c \alpha z - c \beta t)]]
\]

(14)

where the positive constant \( c << 1 \).

Finally, the current generators will be calculated by:

\[
I_g(t) = 2D \int_{i=1}^{\frac{1}{n}} i_i dz, \quad 1 < i < n
\]

(15)

**Calculation of resistances**

The charges in the double layer modify the liquid's conductivity. This means that the closer to the interface, the higher the conductivity is. For that reason, we assume for the calculation of the resistances, that the charge leakage from the solid is produced through a surface of thickness \( \delta_s \) away from the wall (figure 3), in the liquid. The leakage surface is:

\[
S_L \approx 2 \delta_s D
\]

(16)

We suppose that there is only one chemical species involved in the physicochemical reaction at the interface. Equation (17) may therefore express the liquid's conductivity:

\[
\sigma(x, z, t) = \sigma_0 + A \rho(x, z, t)
\]

(17)

\[
A = \frac{e_0}{kT} z D_0
\]

(18)

where \( e_0 \) is the elementary charge, \( k \) the Boltzmann constant, \( T \) the absolute temperature, and \( z \) the ions valence (we assume that positive and negative ions have the same valence).

The potential at the interface is related to the leakage current by:

\[
- S_L \frac{dV}{dz} = \int_{S_L} \frac{1}{A \rho_n(z, t) + \sigma_0} dz \int_{S_L} j dS
\]

(19)

In order to find an expression for the resistances, we calculate the mean value of the space charge density inside the leakage surface \( S_L \):

\[
\rho_n(z, t) = \frac{1}{S_L} \int_{S_L} \rho(x, z, t) dx = \frac{\rho_{wd}}{\rho_{wd}} \rho_n(z, t)
\]

(20)

with:

\[
\rho_{wd} = \rho_{wd} \left[ \tanh \left( \frac{a}{\delta_0} \right) - \frac{\sinh(\frac{a - \delta_0}{\delta_0})}{\cosh(\frac{a}{\delta_0})} \right]
\]

(21)

By using a mean value of the space charge density in the leakage surface, we also assume that the potential is only a function of the axial coordinate \( z \) and time. Hence (19) becomes:

\[
- S_L \frac{dV}{dz} = \int_{S_L} \frac{1}{A \rho_n(z, t) + \sigma_0} dz \int_{S_L} j dS
\]

(22)

As a result, the expression for the resistances is:

\[
R_j(t) = \frac{1}{S_L} \int_{S_L} \frac{1}{A \rho_n(z, t) + \sigma_0} dz
\]

(23)

Integration limits \( z_0 \) and \( z_1 \) of (23) are shown in table 1.

The capacities are estimated for the pressboard-PTFE geometric configuration, and adjusted to match the time constant observed experimentally.

<table>
<thead>
<tr>
<th>node</th>
<th>( z_0 )</th>
<th>( z_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( \frac{L}{2n} )</td>
</tr>
<tr>
<td>2-n</td>
<td>( \left[ \frac{1}{n} \right] + (j-2) )</td>
<td>( \left[ \frac{1}{n} \right] + (j-1) )</td>
</tr>
<tr>
<td>( n+1 )</td>
<td>( \left[ \frac{1}{n} \right] + (j-2) )</td>
<td>( L )</td>
</tr>
</tbody>
</table>

**Results**

In order to resolve the electric circuit, we have developed a finite differences code in Scilab®. The variables in this code are the number of nodes \( n \), which indicates the spatial discretization degree, and the time pass \( dt \), which indicates the temporal discretization degree. The oil parameters, the wall current density coefficient \( K \), the mean flow velocity and the constant \( c \) also need to be introduced.

Figure 4 shows a simulation result made with the parameters presented in table 2, superposed with experimental data.
We believe that this behavior is related to a wall current limitation due to charge accumulation at the interface, and the associated potential rise.

**Discussion**

We have presented an electrical analogy model of a flow electrification phenomenon produced by oil flowing in a pressboard rectangular duct. This model can reproduce fairly well the experimental data, however, it is important to signal that the solution of (7) used is not unique. In addition, future work including a parametric study to determine more precisely the influence of each parameter considered in the model, as well as the introduction in the equations of the accumulated charge in the interface will be done to reproduce more accurately the limitation observed in the generation current and its influence in the other currents.

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**References**


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