Parametric Study by Electrical Analogy: Application to Flow Electrification in Power Transformers

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Abstract— A charge zone named electrical double layer exists at a solid-liquid interface. The liquid flow induces a phenomenon called Flow Electrification: it generates a streaming current (caused by charge convection) and a potential rise in the solid (if this one is insulated from the ground). These potentials may reach values high enough to produce electrical discharges and provoke accidents. Although this phenomenon has been identified a long time ago, its physical description remains unknown (production and displacement of charges, equilibrium, etc.). In this paper we present the results of a parametric study made with an electrical analogy model, which represents the processes taking place when transformer oil flows through a rectangular pressboard duct. The duct’s geometry and the materials were selected to compare some of the numerical results to experimental ones. The facility used to obtain these experimental results was developed some years ago as a part of the research program of Electricité de France and the University of Poitiers.

I. INTRODUCTION

As soon as a liquid gets in contact with a solid, the solid-liquid couple that was initially neutral becomes polarized under physicochemical reactions occurr-
ring at the interface. Such phenomenon leads to a space charge in the liquid, and to a space charge in the solid which can accumulate according to leakage paths [1]. The space charge distribution in the liquid is called electrical double layer.

Convection of the liquid creates a current called Streaming Current, and leads to a continuous charge "separation" process at the interface. When the solid is insulated from the ground, leakage impedances limit the accumulation of these charges at the wall.

In our case, the liquid used is transformer oil, and the solid is a rectangular pressboard duct, which is inserted in a PTFE frame (fig. 1).

Charge leakage takes place towards two stainless steel couplings placed at both extremities of the duct, and insulated from the rest of the loop by PTFE flange couplings.

Two plane electrodes are placed facing the external surfaces of the pressboard duct, beyond 2 mm of PTFE, to measure a mainly capacitive current (Iacc) related to the charge trapped inside the pressboard (accumulation charges).

Typical experimental curves obtained are shown in fig. 2.

A complete description of the facility used to obtain the experimental data, as well as an analysis of static and dynamic equilibriums can be found in [1-2].
In this paper, we model the processes taking place in the facility by an electrical analogy and we present the results of a parametric study.

II. ELECTRICAL ANALOGY MODEL

The charge separation mechanism is modeled with current generators. At any time, their sum is equal to the sum of the accumulation current and both of the leakage currents (fig. 1).

The $\pi$ type array of resistances and capacitors represents the solid (pressboard-PTFE) and the interface.

Equations and hypothesis used in this model are presented hereafter.

Let us consider a rectangular duct of length $L$, whose thickness $2a$ is negligible relative to its width $D$ (fig. 3). Thus, the assumption of two parallel plates is possible.

A redox-type chemical reaction will be assumed to be responsible for the charge separation process at the interface [3–4]. In addition, we consider that our problem may be included among those of weak space charge density. Al-
though this is usually not verified, Touchard has showed that the error made
when the weak space charge approximation is used is often a few percent [5].
This is in the same order of magnitude of experiments accuracy. As a result,
wall current density may be expressed as [4]:

\[ i_w(z,t) = Kf(\xi - \rho_w(z,t)) - Kr \cdot \sigma_i(z,t) \]  

(1)

\[ \xi = \rho_w - \frac{Kr}{Kf} \cdot \sigma_{ad} \]  

(2)

Where \( Kf \) and \( Kr \) are coefficients constant for a given physicochemical
reaction, \( \rho_w \) is the space charge density near the interface and \( \rho_{ad} \) is the space
charge density near the interface for a fully developed double layer, \( \sigma_i \) is the
surface charge density and \( \sigma_{ad} \) is the surface charge density for a fully devel-
oped double layer.

We assume from now on that the time needed for the development of the
diffuse layer is much shorter than the time needed for the development of space
charge at the interface and also shorter than the residence time of a particle in
the duct during the convection. With these hypotheses, the space charge den-
sity in the liquid is given by [6]:

\[ \rho(x,z,t) = \rho_w(z,t) \frac{\cosh(x/\delta_0)}{\cosh(\delta_0/\delta_0)}, \quad \delta_0 = \sqrt{\frac{\varepsilon D_0}{\sigma_0}} \]  

(3)

\( \delta_0 \) being the diffuse layer thickness (also called the Debye length): \( \varepsilon \) is the li-
quid's dielectric constant, \( \sigma_0 \) its bulk conductivity and \( D_0 \) a mean diffusion coeffi-
cient.

The velocity profile for the laminar oil flow is:

\[ U(x) = \frac{3}{2} U_m \left( 1 - \frac{x^2}{a^2} \right) \]  

(4)

With \( U_m \) the mean flow velocity.

The streaming current due to charge convection by the flow is:

\[ I_s(z,t) = \frac{D_a}{2} \int_{-\delta_0}^{\delta_0} \rho(x,z,t) U(x) dx dy \]  

(5)

Charge conservation in the liquid leads to:

\[ \frac{1}{D} \frac{\partial I_s(z,t)}{\partial t} - 2I_s(z,t) = -\int_{-a}^{a} \frac{\partial \rho(x,z,t)}{\partial t} dx \]  

(6)
After replacing $i_w$, $\rho$ and $I_s$ according to (1), (3) and (5), and integrating, (6) becomes:

$$C_1 \frac{\partial \rho_u(z,t)}{\partial z} + C_2 \frac{\partial \rho_s(z,t)}{\partial t} = K_f \lbrack \xi - \rho_u(z,t) \rbrack - K_r \cdot \sigma_s(z,t)$$

(7)

Where $C_1$ and $C_2$ are constants given by:

$$C_z = \delta_0 \tanh \left( \frac{a}{\delta_0} \right) \quad C_1 = \frac{3}{2} \left( \frac{\delta_0}{a} \right)^2 U_w[a - C_2]$$

(8)

Leakage currents are considered to occur at the interface, with the following expression:

$$i_l(z,t) = \frac{1}{\rho_s} \frac{\partial V_s(z,t)}{\partial z}$$

(9)

Here $i_l$ is a linear leakage current density, $\rho_s$ a surface resistivity and $V_s$ the potential at the interface.

Charge conservation at the interface leads to:

$$i_s(z,t) + \frac{\partial i_l(z,t)}{\partial z} + \frac{\partial \sigma_s(z,t)}{\partial t} = 0$$

(10)

The Potential at the interface $V_i$ and the surface charge density $\sigma$ are related through:

$$V_i(z,t) = [\varepsilon E_x(a,z,t) + \sigma_s(z,t)] \frac{1}{C_3} \quad C_3 = \frac{\varepsilon_x E_x}{\varepsilon_1 w_p + \varepsilon_p w_T}$$

(11)

$$E_x(a,z,t) = \frac{C_2}{\varepsilon_x} \rho_u(z,t) + \frac{\varepsilon_0}{\varepsilon_1} \int_0^z \frac{\partial^2 V(x,z,t)}{\partial z^2} dx$$

(12)

$\varepsilon_x$ and $\varepsilon_p$ are the PTFE and pressboard dielectric constants. $w_T$ and $w_p$ are their thicknesses. $E_x(a,z,t)$ is the component of the electric field normal to the interface. From geometrical considerations we can assume that the second term in the right hand side of (12) is negligible with respect to the first one, thus:

$$\sigma_s(z,t) = C_3 V_i(z,t) - C_2 \rho_u(z,t)$$

(13)

Equations (1), (7), (10) and (13) are combined and discretized to obtain the system of equations that can be represented by the electrical model shown in fig. 1.

A finite differences code was developed in Scilab® to solve this system of linear partial differential equations. The variables in this code are the number of nodes $n$, which indicates the spatial discretization degree, and the time pass
\( dt \), which indicates the temporal discretization degree. The oil parameters (\( \sigma_0 \) and \( \varepsilon \) are known, \( D_0 \) was estimated), the wall current density coefficients \( K_f \) and \( K_r \), and the mean flow velocity also need to be introduced. The simulation always starts with a static, fully developed electrical double layer. As a result, at the beginning, \( \rho_w \) is equal to \( \rho_{wd} \) and the surface potential \( V_s \) is null, all along the duct. Then, at a determinate time, flow can be stopped to simulate the "discharge".

### Table 1: Simulation Parameters

<table>
<thead>
<tr>
<th>( \sigma_0 ) (S/m)</th>
<th>( D_0 ) (m²/s)</th>
<th>( \varepsilon ) (F/m)</th>
<th>( U_0 ) (m/s)</th>
<th>( K_f ) (m/s)</th>
<th>( K_r ) (1/s)</th>
<th>nodes/cm</th>
<th>( dt ) (s)</th>
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<td>0.4</td>
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</table>

### III. RESULTS

With the parameters presented in table 1, we obtained the results shown in figures 4, 5 and 6. In fig. 4 we present the simulated currents: the model reproduces fairly well the experimental results.

The positive pick observed (in fig. 2) on the downstream leakage current at the beginning of the flow, has been found to be related to an influence of the streaming current development over the downstream leakage current measurement: positive charges, convected by the flow, are "seen" by the downstream stainless steel coupling, connected to the downstream pico-ammeter. This experimental issue has not been simulated in this work. The difference shown between the streaming current and the sum of the accumulation current and both of the leakage currents, is due to the input impedance of the pico-ammetters, cables capacities and PTFE flange couplings capacities, which in turn have been considered in the model.
Fig. 5. Temporal and spatial evolution of the space charge at the interface ($\rho_w$)

Fig. 5 shows the evolution of the space charge at the interface, at $t=0$s we observe a constant value along the duct, but as time passes the space charge evolves to the dynamic equilibrium distribution.

Fig. 6. Surface potential ($V_s$) development

We can see in fig. 6 that surface potential evolves towards an almost symmetrical distribution with respect to the center of the duct. This brings us to a more thorough analysis of fig. 4. Although it reproduces well the experimental data, when we observe both leakage currents, their difference (leakage is more important towards the upstream side) is not important enough compared to experimental results. This would mean that the dissymmetry in the "generation" is not enough to reproduce this phenomenon. In view of that, we have simulated the same problem, but with a surface resistivity inversely proportional to the generation (the total resistance: the sum of all the leakage resistances, was kept constant). The result is shown in figs. 7 and 8.
With this hypothesis, the difference between leakage currents is greater and simulation has a better agreement with experimental results.

The surface potential (V_s) development. Surface resistivity inversely proportional to generation

Although in our problem surface resistivity seems to be modified by flow electrification, this behavior's physical explanation still needs to be found. For that reason, the results shown hereafter were calculated with a constant surface resistivity.

In fig. 9 we present the streaming current's temporal evolution for four ducts of different lengths. As a first observation, we can say that for a given mean velocity, streaming current will be greater the longer the duct is. Yet this is only true until a critical length is reached, beyond which increasing the length will not result in an increase in the streaming current. This can be understood by observing fig. 10. For a given mean flow velocity, a longer duct will allow a more important development of the double layer, and thus a greater stream-
ing current. However, once the double layer is fully developed, the quantity of charges convected can no longer be increased because the wall current becomes null for the rest of the duct.

Fig. 9. Streaming current as a function of time, for different duct lengths

Fig. 10. Space charge at the interface as a function of position (non-dimensional) for different duct lengths at \( t=59.6s \)

Fig. 11 shows the surface potential distribution for four duct lengths. For a given time, increasing the duct's length will increase less and less the potential's extremum. It is important to say, though, that increasing the length will increase the time needed to attain electrical equilibrium (the circuit's time constant is multiplied). Therefore when a duct of a given length has reached the generation/leakage equilibrium, a longer duct will continue to accumulate charge and increase its potential. This can be seen in fig. 12, where simulation was continued until \( t=5000s \). Over \( t=400s \), the potential distribution doesn’t change for the short duct (curves appear superposed), meanwhile it continues to increase for the long one.
Finally, in figs. 13, 14 and 15 we present the influence of mean velocity. Fig. 13 shows that the streaming current's pick value at the beginning increases almost proportionally with velocity, but the dynamic equilibrium value reached increases less and less. This behavior is typical for a non-fully developed double layer. As it can be seen in fig. 14, an increase in velocity will produce a less developed double layer, so the increase in streaming current will be less important. This is not the case at the beginning, when the double layer is considered to be fully developed.
Fig. 13. Streaming current as a function of time, for different mean flow velocities

Fig. 14. Space charge at the interface as a function of position for different flow velocities at $t=59.6s$

Surface potential distribution is presented in Fig. 15, and we can observe that the increase in its extremum is less and less important.

Fig. 15. Surface potential as a function of position for different flow velocities at $t=59.6s$
IV. DISCUSSION

We have presented an electrical analogy model of a flow electrification phenomenon produced by oil flowing in a pressboard rectangular duct. This model can reproduce fairly well the experimental data; however, it would seem necessary to take into account a non-uniform distribution of surface resistivity. The first results of a parametric study have been presented and discussed: they show typical behavior of flow electrification produced by a non-fully developed electrical double layer. In future work, the influence of oil parameters will be studied and we will continue to search for an accurate model for surface resistivity distribution.

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REFERENCES


