Fracture-Energy-based interface theory for Fiber Reinforced Concrete failure analyses

A. Caggiano¹, G. Else² and E. Martinelli³

ABSTRACT: The present paper deals with the numerical analysis of fiber-reinforced concrete (FRC) by using special “interface” elements for modeling the nonlinear fracture behavior under general loading conditions. After a brief overview about the basic features of the numerical model, some experimental results currently available in the scientific literature are considered for its validation. Finally, the capabilities and shortcomings of the proposed model for FRC failure analyses are evaluated.

1 INTRODUCTION

Concrete is one of the most common materials for civil structures. It is characterized by rather good mechanical properties for members in a predominantly compressive stress state, while weaker performances are expected under more general load conditions, generally requiring a steel reinforcement to resist tensile and shear stresses. The weak mechanical properties of plain concrete under those stress states can be mitigated by adding short fibers (made out of either steel or plastics) into the cementitious matrix. Several models, analytical formulations and numerical techniques have been developed to represent the mechanical behavior of fiber-reinforced concrete (FRC). Among them, various elasto-plastic models are presented into the scientific literature (e.g. Hu et al. 2003). Other authors modeled the FRC material by means the continuum damage mechanism (CDM) theory (Li and Li, 2001). A rather innovative approach inspires the so-called microplane model (Vrech et al. 2010). Moreover, an analytical model to predict the tensile behavior and the moment-curvature and load-deflection properties of steel fiber reinforced beams is proposed by Lim et al. (1987). A meso-mechanical approach has been recently utilized for overcoming the weaknesses affecting concrete models based on continuum-mechanics. The so-called meso-mechanical approach can lead to several proposals such as lattice models (Lilliu and van Mier 2003), particle models (Zubelwicz and Bazant 1987) and continuum meso-models (Lopez et al. 2008). These approaches provide a much more powerful and physically-based description of the material behavior, modeling with special accuracy the fracture processes and the mechanical properties of plain and fiber reinforced concretes. The apparent macroscopic behavior observed is a direct consequence of the more complex meso- or micro-scopic phenomena that take place at the level of the material heterogeneities. Furthermore, the principal disadvantages of these approaches are related, on the one hand, to the higher computational cost, and, on the other hand, to the huge number of variables involved in the model formulation. The approach proposed in the present paper may be included in the latter group, and represents the extension to FRC materials of a 2-D model for meso-mechanical analysis of concrete using fracture-based zero-thickness interface elements. The original interface model (Carol et al. 1997) is reformulated to include both bond-slip behavior and

¹ Ph.D. Student, Department of Civil Engineering, University of Salerno (Italy)
² Professor of Structural Mechanics, CONICET, University of Buenos Aires (Argentina)
³ Assistant Professor, Department of Civil Engineering, University of Salerno (Italy)
dowel interaction between steel fibers and the surrounding cement matrix, based on the composite mixture theory, also utilized by Oliver et al. (2008). The model is calibrated by using some results of tensile tests, already available in the scientific literature. A final discussion about the capability of the model for simulating the behavior of FRC specimens with different geometrical and mechanical parameters is proposed. In particular, the effect of different volumetric ratio of fibers is simulated.

2 OUTLINE OF THE FORMULATION OF THE NUMERICAL MODEL

The formulation of a theoretical model based on Fracture Energy and Plasticity is outlined in the following. The five subsections focus on the key aspects of the model.

2.1 Basic assumptions

Fracture analysis of FRC at the meso-level follows the approach proposed by Vonk (1992) and further developed by Lopez et al. (2008). While continuum concrete elements are assumed as linear elastic, the non-linear behavior of plain and fiber-reinforced concrete is localized in the interface elements. According to the assumptions of the mixture theory, the composite is considered as a continuum in which each infinitesimal volume is ideally occupied by all the components. In the plane of the interface, all the components are subjected to the same strain field and the corresponding composite stresses are given by the weighted sum of the stresses on each component. Let $u$ be the interface relative displacement vector. The corresponding value of the axial displacement of the fiber (in $n$ direction) is given by $u_N = u \cdot n$, while in the transversal direction, $u_T = u \cdot n_T$, where $n$ and $n_T$ are the two unit vectors parallel and orthogonal to the direction of the generic fiber (Figure 1). The axial and transverse components of the (average) strain developed in the generic fiber can be defined as follows:

$$\varepsilon_N = \frac{u_N}{l_f}, \quad \gamma_T = \frac{u_T}{l_f}.$$ (1)

![Figure 1. Schematic configuration of a joint crossed by one fiber.](image)

2.2 Constitutive meso-mechanical model

The present interface joint constitutive model is formulated in incremental form, as usual within the flow theory of plasticity. According to the mixture theory, the rate of the stress vector $\dot{\sigma}$ in the interface plane, depends on the volumetric fraction, $\rho$

$$\dot{\sigma} = \rho \dot{\epsilon} + \xi \rho \dot{\sigma}, \quad (\dot{\varepsilon}_n + \xi \rho \dot{\gamma}_T).$$ (2)

being $\xi = 1 - a \rho$ (with “$a$” model parameter) a reductive coefficient that considers the minor effectiveness of a single fiber as the reinforcement contents increase. The superscripts m and f refer to concrete matrix and fibers, respectively, while the superscript t represents the transposition operation for tensors. The incremental stress-displacement of the proposed joint model can be expressed in compact form as
\[ i = E^\sigma \cdot \dot{u} , \]  

where the constitutive tangent matrix \( E^\sigma \) is

\[ E^\sigma = \rho^\sigma C^\sigma + \xi^\sigma \rho^\sigma \frac{E^\sigma_p}{l_f} \mathbf{n} \cdot \mathbf{n} + \bar{G}^\sigma \mathbf{n} \cdot \mathbf{n} \cdot . \]  

with the following symbolic definition of the tensors involved in eq. (4):

\[ C^\sigma = \frac{\partial t}{\partial u} , \quad E^\sigma = \frac{\partial \sigma}{\partial \varepsilon} , \quad G^\sigma = \frac{\partial \tau}{\partial \gamma} . \]  

The three following aspects can be considered in defining the parameters controlling the process fracture propagation in FRC:

- fracture energy-based joint constitutive law, discussed in subsection 2.3;
- fiber bond-slip effects, described in subsection 2.4;
- dowel action of short fibers crossing cracks, addressed in subsection 2.5.

### 2.3 Constitutive meso-mechanical model

The interface model, originally proposed by Gens et al. (1988), is summarized. The elasto-plastic formulation of the interface model in rate form is defined by

\[ \dot{u} = \dot{u}'' + \dot{u}''' , \]  
\[ \dot{u}''' = C^{-i} i , \]  
\[ i = C (\dot{u} - \dot{u}'') , \]

where \( \dot{u}'' = [\dot{u}, \dot{v}] \) is the rate vector of relative displacements, decomposed into the elastic and crack opening components \( \dot{u}'' \) and \( \dot{u}''' \), respectively. The tensor \( C \) defines a fully uncoupled normal/tangential elastic stiffness at the interface:

\[ C = \begin{bmatrix} k_v & 0 \\ 0 & k_t \end{bmatrix} . \]

The yield-loading condition of the interface constitutive model is defined as follows (Figure 2)

\[ F(\sigma, \tau) = \tau^2 - (\sigma - \chi \tan \phi)^2 + (\sigma - \chi \tan \phi)^2 , \]

where \( \tau \) and \( \sigma \) are the interface stresses on the interface joint element. The tensile strength \( \chi \) (vertex of the hyperbola in Figure 2), the shear strength \( \epsilon \) (cohesion strength) and the internal friction angle \( \phi \) are model parameters.

![Figure 2. Initial yielding surface for the interface joint element.](image)

Two limit situations can be easily recognised in eq. (8):

- (a) cracking under pure tension, with zero shear stress (Mode I), when the yield surface is reached along the horizontal axis;
(b) cracking under shear and very high compression, when the yield surface is reached in its asymptotic region, where the hyperbola approaches a Mohr-Coulomb criterion. The last one is called "asymptotic Mode II (or Mode IIa)". The evolution of fracture process is driven by the cracking parameters \( \chi \) and \( c \), which depends on the energy release during interface degradation \( W_{cr} \). Further details can be found in Carol et al. (1997).

2.4 Fiber bond-slip interaction

The uniaxial behavior of the steel fiber is approximated by means of a simple 1D elasto-plastic model. The following set of equations is considered

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p, \quad (11)
\]

\[
\dot{\varepsilon}^e = \frac{\sigma_f}{E_f}, \quad (12)
\]

\[
\dot{\sigma}_f = E_f (\dot{\varepsilon} - \dot{\varepsilon}^e), \quad (13)
\]

where the rate of the axial fiber strain \( \dot{\varepsilon} \) is decomposed into an elastic and a plastic part \( \dot{\varepsilon}^e \) and \( \dot{\varepsilon}^p \), respectively. \( E_f \) represents an equivalent uniaxial elastic modulus including the uniaxial response of the steel and the bond-slip effect of the short reinforcement, and \( \sigma_f \) is the rate of bond-slip axial stress of the steel fiber. The yield criterion, in tension as well as in compression, is represented by the following expression

\[
F_f = \sigma_f - (\sigma_{y,f} + Q_f), \quad (13)
\]

where \( \sigma_{y,f} \) is the elastic limit. The evolution, in the post-elastic regime of the 1D surface is driven by the stress-like internal variable \( Q_f \), given in incremental form as

\[
\dot{Q}_f = \dot{\lambda}_f H_f, \quad (14)
\]

with

\[
\dot{\varepsilon}^{px} = \dot{\lambda}_f \frac{\partial F_f}{\partial \sigma_f} = \dot{\lambda}_f \text{sign} [\sigma_f], \quad (15)
\]

representing the plastic flow law, \( \dot{\lambda}_f \) is the non-negative plastic multiplier and \( H_f \) is the hardening/softening modulus. The incremental stress-strain relationship is

\[
\dot{\sigma}_f = E_f^{pp} \dot{\varepsilon}^p, \quad (16)
\]

where the elasto-plastic tangent modulus \( E_f^{pp} \) takes the two following distinct values, (Figure 3)

\[
E_f^{pp} = \begin{cases} E_f & \rightarrow \text{Elastic Response} \\ E_f \frac{1}{E_f/H_f+1} & \rightarrow \text{Elasto-Plastic regime} \end{cases}
\]

\[
(17)
\]

It is assumed that the fiber strain \( \varepsilon \) is decomposed in two additive parts, one due to the intrinsic fiber uniaxial deformation \( \varepsilon_s \) and another one associated with the interface debonding \( \varepsilon_d \). Assuming a serial model constituted by the fiber and the fiber-mortar joint the corresponding total flexibility

\[
\frac{1}{E_f} = \frac{1}{E_s} + \frac{1}{E_d}, \quad (18)
\]
where $E_s$ and $E_d$ are the steel Young modulus and an equivalent elastic modulus of matrix-fiber interface, respectively.

In this context, and to complete the bond-slip axial constitutive models presented by the Eqs. (11) to (18), the following material parameter are defined

$$\sigma_{y,s} = \min(\sigma_{y,s}, \sigma_{y,d}), \quad (19)$$

$$H' = \begin{cases} H' & \text{if } \sigma_{y,s} < \sigma_{y,d} \\ H' & \text{if } \sigma_{y,s} \geq \sigma_{y,d} \end{cases} \quad (20)$$

in which $\sigma_{y,s}$ and $\sigma_{y,d}$ are the material yield stress and the equivalent interface elastic limit, respectively; while the super-indices $s$ and $d$ refer to steel and debonding, respectively. The parameters $E_d$, $\sigma_{y,s}$ and $H'$ required for the bond/slip model characterization can be calibrated from a simple pull-out test (Oliver et al. 2008).

### 2.5 Dowel action of fibers

The dowel effect of fibers crossing cracks in mortar is simulated by the joint model through a 1D shear stress-strain elasto-plastic constitutive model, similar to the previously mentioned one for the axial stress-strain. In this case, the following equations are utilized

$$\dot{\gamma} = \dot{\gamma}^e + \dot{\gamma}^p \quad , \quad (21)$$

$$\dot{\gamma}^e = \frac{\dot{\tau}_f}{G_f} \quad , \quad (22)$$

$$\dot{\tau}_f = G_f(\dot{\gamma} - \dot{\gamma}^e) \quad , \quad (23)$$

being $\gamma$ the rate of the shear fiber strain, which is decomposed in eq. (21) into a elastic and a plastic part. $G_f$ represents the shear modulus, while $\dot{\tau}_f$ is the rate of dowel shear stress of the interaction between fiber-matrix. The model is completed with:
- a yielding criterion, similar to Eq. (13);
- a hardening/softening law, similar to Eq. (14).

The incremental shear stress-strain relation, can also be written as

$$\dot{\tau}_f = G_f^p \dot{\gamma} \quad , \quad (24)$$

where the tangent shear modulus $G_f^p$ can be defined in eq. (17) for $E_f^p$, introducing a hardening-softening modulus of the uniaxial dowel model, commonly assumed as $H'_{dow}=0$. In the present model the dowel effect is simulated considering the well-known Winkler model for elastic beams on elastic foundation. Details about the implementation of that model within the general framework of the present nonlinear procedure can be found in Vrech et al. (2010).

### 3 EXPERIMENTAL COMPARISONS AND VALIDATION OF THE NUMERICAL MODEL

In this section the presented model will be validated by using of the results of experiments on steel fiber-reinforced concrete SFRC specimens tested in pure tension. The
element patches shown in Figure 4 are considered for evaluating the predictive capability of the cracking model. In particular, one interface element connects two 4-node plane stress isoparametric elements. Different fiber contents can be considered. The number of fibers is one of the model parameters and can be directly evaluated considering the volume content and the properties of fibers. Figure 4 shows the possible arrangements of fibers throughout the potential fracture line, depending on the number. Every fiber is assumed to cross the potential fracture line at mid-length (i.e. at \( f_{l} / 2 \)).

The results of tensile tests on SFRC specimens presented by Li et al. (1998) are firstly considered. Two different kinds of fibers, both with hooked ends, have been utilized therein:  
a) steel fibers “Dramix” (diameter \( d=0.5 \text{ mm} \), \( l=30 \text{ mm} \), density \( \rho=7.8 \text{ g/cm}^{3} \), tensile strength \( f_{u}=1.2 \text{ GPa} \), \( E=200 \text{ GPa} \));  
b) steel fibers “Harex” (with arched cross section of area \( 2.2\times 0.25 \text{ mm}^{2} \), \( l=32 \text{ mm} \), density \( \rho=7.8 \text{ g/cm}^{3} \), tensile strength \( f_{u}=0.81 \text{ GPa} \), \( E=200 \text{ GPa} \)).  
An indirect identification of the numerical model presented in section 2 has been performed for calibrating the model on the experimental results obtained on FRC specimens with the above two types of fibers. In particular, the set of those parameters (i.e., the equivalent elastic modulus of fibers, \( E_{d} \), the equivalent interface elastic limit, \( \sigma_{y,d} \), and the other parameters mentioned in the section 2), collected in the vector \( q \), have been derived by the following least-square procedure:

\[
\bar{q} = \arg \min_{q} \left[ \sum_{i} (\sigma_{\text{exp},i}^{q} - \sigma_{\text{exp},i})^{2} \right],
\]

where the \( \sigma_{\text{exp},i}^{q} \) is the stress value derived by the theoretical model for the strain value \( \varepsilon_{\text{exp},i}^{q} \) and the set of parameters \( q \), while \( \sigma_{\text{exp},i} \) is the corresponding stress value. The sum of the squared differences between the experimental observation and the corresponding theoretical value is extended to all the \( n \) experimental measures available in terms of axial strain-stress couples.

The least-square calibration procedure has been carried out twice for calibrating the numerical model on the experimental tests on specimens made out of the two different kinds of fibers described above. In particular, both those two different calibrations have been performed with reference to the \( n \) \((\varepsilon_{\text{exp}}, \sigma_{\text{exp}})\) couples obtained in the cases of plain concrete \((\rho=0)\) and highest volume content of fibers \((\rho=0.06)\). For the sake of brevity, the complete results of the two calibration procedures performed on the experimental tests presented by Li et al. (1998) are not completely reported herein. However, Table 1 outlines the values of the key parameters related to the fibers, and directly affected by their nature and geometry.

<table>
<thead>
<tr>
<th>Type of fibers</th>
<th>( E_{d} ) [GPa]</th>
<th>( \sigma_{y,d} ) [MPa]</th>
<th>( k_{e} ) [N/mm(^{2})]</th>
<th>( k_{t} ) [N/mm(^{2})]</th>
<th>( c ) [MPa]</th>
<th>( \chi ) [MPa]</th>
<th>( \tan \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dramix</td>
<td>200</td>
<td>2580</td>
<td>1000</td>
<td>1000</td>
<td>7.0</td>
<td>4.0</td>
<td>0.60</td>
</tr>
<tr>
<td>Harex</td>
<td>200</td>
<td>1741.5</td>
<td>1000</td>
<td>1000</td>
<td>7.0</td>
<td>4.0</td>
<td>0.60</td>
</tr>
</tbody>
</table>
The experimental tests and the simulations obtained after the calibration procedure, are reported in figure 5a and b, for the specimens reinforced with “Dramix” and “Harex” fibers, respectively. In particular, in both cases the curves related to the cases of both plain concrete and highest fiber content approximate rather well the corresponding experimental observation as a direct result of the least-square calibration described by equation (25). Moreover, the same figure shows that the calibrated model results in a very good simulation of the experimental results even in the two cases of specimens with intermediate values for the volume content of fibers (namely, 0.02 and 0.03 approximately), obtained just changing the fiber content in the identified above model.

Moreover, the same model identified for the specimens with “Dramix” fibers has been also utilized for simulating the results of similar tensile tests reported by Li and Li (2001). Two different values have been considered in the experimental program for fiber content, ranging from 6% to 8%. Figure 6 reports both the experimental results and the numerical simulation for those two cases, along with the reference result derived by a specimen made out of plain concrete.

Beyond the general accuracy of the model and, consequently, the soundness of its assumptions and formulation, the above figure 5 and 6 emphasize a clear phenomenological behavior of FRC, depending on the fiber content and quality. A progressive transition of the stress-strain relationship form the typical softening behavior of plain concrete toward a more ductile behavior can be observed for the various values of fiber content, ranging from 2% to 8% in the analyzed cases. The two diagrams in figure 5 points out the different behavior fiber with different transverse sections, yet both hooked at the end. The higher efficiency (in terms of both strength and ductility) shown in figure 5a probably depends on the fact that the volume of “Dramix” fibers is significantly smaller (almost three times) than the one of “Harex” ones (figure 5b). Consequently, a significantly higher number of fibers are present in the first specimens resulting in a more homogeneous material.
CONCLUSIONS

A numerical model based on the described interface elements has been presented in the present paper. The model is particularly suited to model the stress-strain behavior of FRC in tension, capturing the oriented nature of cracks and the fiber effect in bridging their opening, by means of two main mechanisms, such as bonding strength and dowel action in axial and transverse direction, respectively.

The identification procedure needed for calibrating the mechanical parameters involved in the model formulation has been outlined with reference to two series of experimental results available in the scientific literature and obtained by tensile tests of FRC specimens. The calibration has been performed in terms of load-displacement curves with respect to the experimental results of only two specimens of the same batch. In particular, the specimens made out of plain concrete and the one with the highest fiber contents have been considered in the identification procedure. Nevertheless, the identified model led to very good simulations of the observed behavior in all the other specimens featuring intermediate values contents of the fibers content.

Further developments are currently ongoing in order to improve the modeling of fiber-concrete bonding behavior. Finally, the assessment and identification of the present model to simulate the behavior of concrete members under more general load conditions and induced stress states is another objective of the future researches.

REFERENCES


