BOND BEHAVIOR OF FRP STRIPS GLUED ON CONCRETE: A NOVEL ZERO-THICKNESS INTERFACE MODEL

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Abstract. This paper presents an innovative approach for simulating the bond interaction of Fiber Reinforced Polymer (FRP) strips glued to a concrete substrate. It is actually based on employing zero-thickness interface elements for connecting the FRP strip to the concrete substrate. The mechanical formulation of those elements, which have been already adopted in various fields of computational mechanics, is firstly outlined in the paper. Then, their application for simulating the behaviour observed in pull-out tests on FRP strips glued to concrete is proposed. The key mechanical parameters which define the relevant aspects of interface elements are calibrated by means of relevant experimental results currently available in the scientific literature. Finally, a series of numerical analyses carried out are proposed. They are intended at pointing out the potential of the proposed model which is formulated within a more general mechanical framework, without assuming any “a priori” bond-slip relationship, as usual in the scientific literature on this subject. Finally, the possible extension of the model for simulating more general load conditions (e.g., cyclic actions) is among the very next developments of the present research.
1 INTRODUCTION

FRP materials are nowadays largely used as efficient solution for retrofitting existing concrete (or even masonry) structures. However, the possible premature failure which often develops throughout the FRP-to-concrete adhesive interface generally controls the effectiveness of FRP applications and plays a crucial role for the stress transfer from FRP strips to concrete substrate. This premature failure is generally referred as “debonding” (Yao et al., 2005).

The mechanical properties of the adhesive interface can be investigated by means of pull-out tests which can be carried out by assuming different experimental layouts, such as single (Chajes et al., 1996; Taljsten, 1996) or double pull-push shear test (Sato et al., 2001), single delamination test fixing the back side of the specimen (Ferracuti et al., 2008), intermediate crack-induced debonding schemes (Teng et al., 2003), etc.

Accurate characterisation of the mechanical parameters, interface performance and failure of FRP-to-concrete bond process are essential for design of structures externally bonded with FRP composites. A large number of contributions deal with the prediction of the plate end debonding failure modes. Closed-form solutions (Caggiano et al., 2011; Cornetti and Carpinteri, 2011; Faella et al., 2008; Yuan et al., 2004) as well as available numerical models (Lu et al., 2006; Ferracuti et al., 2006) are generally aimed at predicting the ultimate axial load resulting in FRP debonding and the key features of the actual failure mode.

The present paper formulates a novel fracture-energy-based model for simulating the bond behaviour of FRP strips glued to concrete substrates. A zero-thickness element is actually considered for describing the mechanical response of the adhesive interface and its fracture processes under pure “mode II” (namely, slipping mode). The comparison between those simulations and the available solutions (either, analytical or numerical in nature) currently available in the scientific literature and generally based on assuming an “a priori” bond-slip relationship is the key aim of this paper and the fundamental motivation for the proposed formulation.

After a brief literature review, section 2 outlines the general formulation of the elasto-plastic models featuring fracture energy-based softening of the shear strength parameter. It present the possible application of zero-thickness interface elements for simulating the bond behaviour of FRP strips glued to concrete.

Section 3 covers the aspects related to the numerical validation of the proposed model with the aim of demonstrating the predictive capabilities of the proposed formulation against experimental tests.

Section 4 mainly analyses the mechanical response in terms of the FRP axial strain vs. bond length and the load vs. displacement curve provided by the present approach with the aim to compare the same response with similar ones obtainable with a simpler bilinear model proposed by the same authors (Caggiano et al., 2011), in which the same fundamental mechanical parameters on both model will be fixed: i.e., shear strength, initial elastic stiffness and the fracture energy in mode II.

2 NUMERICAL MODEL

This section presents a finite element model aimed at simulating the fracture behaviour observed in pull-out tests carried out on FRP strips glued on concrete substrates. A simplified 1D-analysis is proposed by modelling the FRP strip by means of simple elements which are connected to the substrate by means of zero-thickness fracture-based interface joints (Fig. 1).

The behaviour of the FRP strip can be considered as linear elastic. Thus, the strip is discretised by means of iso-parametric rod elements, while the bond between FRP and concrete
are modelled by using the proposed inelastic interface formulation detailed in the following. Moreover, the concrete block is considered as a rigid body for the sake of simplicity.

The formulation of the above mentioned interface elements is based on the usual additive decomposition of the infinitesimal slip rate, $\dot{s}$, between the considered strip and concrete substrate

$$\dot{s} = \dot{s}^e + \dot{s}^p$$  \hspace{1cm} (1)  

being $\dot{s}^e$ and $\dot{s}^p$ its elastic and plastic parts, respectively. Based on Eq. (1), the elastic constitutive relationship can be defined as follows

$$\dot{\tau} = k_E \dot{s}^e \Rightarrow \dot{\tau} = k_E (\dot{s} - \dot{s}^p)$$  \hspace{1cm} (2)  

In the above equation $\dot{\tau}$ is the rate of interface shear stress and $k_E$ the elastic stiffness of the FRP-concrete bond.

Since a fracture process in mode-II is assumed for the sake of simplicity, a one-dimensional yield criterion, denoted as $f(\tau, \kappa)$, controls the post-elastic behaviour, whose size and shape depend on a state variable $\kappa$. The failure criterion is defined in the stress space in terms of the transferred shear stress as follows

$$f(\tau, \kappa) = \tau^2 - \tau_y^2 \leq 0$$  \hspace{1cm} (3)
where $\tau_y$ is the shear bond strength.

After the yield strength is reached, a progressive softening behaviour can be defined by the softening level parameter $\kappa = \frac{w_{sl}}{G_f}$, which represents the ratio of the plastic work made during the inelastic slips and the available fracture energy, being this latter a key model parameter. An even more general definition could be assumed for $\kappa$ by introducing an exponent in the ratio which makes faster or slower the fracture process as described by the fracture work-to-energy ratio $\frac{w_{sl}}{G_f}$. However, the influence of this parameter is neglected in this study for the sake of brevity and will be examined in the future developments of the proposed formulation.

Thus, as a matter of fact, the softening behaviour is modelled by reducing the maximum shear stress $\tau_y$ as follows:

$$\tau_y = \tau_{y,0} (1 - \kappa) \quad (4)$$

where $\tau_{y,0}$ is the maximum shear strength representing a material parameter of the model.

The inelastic material response is governed by the following flow rule

$$\dot{s}^p = \dot{\lambda} \frac{\partial f}{\partial \tau} = 2 \cdot \dot{\lambda} \cdot \tau \quad (5)$$

where $\dot{\lambda}$ is the plastic multiplier which can be determined in each analysis step by applying the well-known Kuhn-Tucker consistency condition

$$\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} \cdot f = 0. \quad (6)$$

Considering the differentiated form of the incremental shear-slip relationship, the following expression can be obtained

$$\Delta \tau = k_E \cdot \left( \Delta s - \Delta \lambda \frac{\partial f}{\partial \tau} \right) \Rightarrow d\Delta \tau = k_E \cdot \left( d\Delta s - d\Delta \lambda \frac{\partial f}{\partial \tau} - \Delta \lambda d\frac{\partial f}{\partial \tau} \right) \quad (7)$$

The global constitutive matrix $k_G$ of the non-linear FE problem is built by adopting the local algorithmic tangent operator, $k_{tan}^{ep}$, derived by the linearised tangential form of the Eq. (2) and given in the following expression

$$d\Delta \tau = k_{E,2} \cdot \left( 1 - \left( \frac{\partial f}{\partial \tau} \right)^2 \cdot k_{E,2} + \Delta \lambda \frac{\partial f}{\partial \tau} \left( \frac{\partial f}{\partial \tau} \frac{\partial \kappa}{\partial \sigma} \cdot \frac{\partial \kappa}{\partial \sigma} \cdot k_{E,2} \right) \right) \cdot d\Delta s \quad (8)$$

where $k_{E,2}$ is the modified elastic stiffness obtained by adopting of the Hessian operator and $H$ is the effective softening modulus. Further details about the solution of the incremental formulation outlined above can be found in Kang and Willam (1999) and Carol et al. (1997).

### 3 NUMERICAL SIMULATIONS

The proposed modelling technique is applied in this section for simulating the behaviour observed in pull-out tests on FRP strips glued on concrete. Since the interface behaviour is often modelled by assuming “a priori” non-linear bond-slip relationships, the main aim of the proposed model is to investigate the problem under consideration within the framework of the
more general theory of fracture mechanics implemented through the mentioned zero-thickness joints.

This section presents the model predictions by using the experimental results reported by Chajes et al. (1996) on four tests involving specimens characterised by different bonding lengths ranging between 50.8 mm (2 inches) and 203.2 mm (8 inches).

The same set of experimental tests have already been analysed by Faella et al. (2009) with the aim of identifying the mechanical parameters of a bi-linear bond-slip curve assumed “a priori”. In particular, a set of relevant mechanical parameters (i.e. \( \tau_{y,0} \) and \( G_f \)) has been identified for each experimental test. As a matter of principle, they can be slightly different as a result of the natural randomness of the mechanical properties of concrete, even though, in fact, the identification procedure leads to the rather similar values summarised in Table 1. The same table reports also the number of rod elements employed in the numerical simulations proposed in the present paper for each specimens.

![Figure 3: Simulation of axial strain distribution (the dots represents the corresponding experimental measures) and force-slip relationship: Chajes et al. (1996) 2 – inches.](image)

![Figure 4: Simulation of axial strain distribution (the dots represents the corresponding experimental measures) and force-slip relationship: Chajes et al. (1996) 4 – inches.](image)

Moreover, an effective Young modulus \( E_f = 110.4 \, GPa \), the width \( b_f = 25.4 \, mm \) and thickness \( t_f = 1.016 \, mm \) have been directly assumed in the analyses on the bases of the experimental report (Chajes et al., 1996).

Figs. 3 to 6 show the numerical results obtained in the numerical simulations carried out on the models of the four tested specimens. Those figures reports both the strain distribution...
throughout the FRP strip length (for different load levels) and the resulting relationship between the applied load and the maximum interface slip measured at the loaded end, up to the onset of the interface debonding process.

The distributions of axial strains developed in the FRP strips throughout their bonded length are also compared with the corresponding experimental results observed by Chajes et al. (1996). The comparisons between numerical results and experimental measures confirm the significant predictive capacity of the proposed model. It is worth noting that this result has not been obtained by “calibrating” the parameters of the zero-thickness interface which, in fact, have been assumed on the bases of the results obtained in Faella et al. (2009) with reference to a “bi-linear” bond-slip relationship.

Finally, the simulations carried out by means of the zero-thickness fracture-based model can be also represented in terms of shear-stress-interface-slip couples obtained in a given Gauss

<table>
<thead>
<tr>
<th>τ_{y,0} [MPa]</th>
<th>k_{E} [MPa/mm]</th>
<th>G_{f} [N/mm]</th>
<th>n_{rod} = n_{joint}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 inches</td>
<td>8.06</td>
<td>157.00</td>
<td>1.31</td>
</tr>
<tr>
<td>4 inches</td>
<td>7.03</td>
<td>542.28</td>
<td>1.98</td>
</tr>
<tr>
<td>6 inches</td>
<td>6.32</td>
<td>494.63</td>
<td>1.39</td>
</tr>
<tr>
<td>8 inches</td>
<td>6.15</td>
<td>519.74</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Table 1: Material properties adopted in the numerical simulation against experimental data by Chajes et al. (1996).
point of the presented numerical model. Those relationships are represented in Fig. 7 by four thin solids lines for the various models referring to the four tested specimens. The corresponding experimental points, obtained approximately by the experimental measures in terms of axial strains, are also reported in the same figures. Fig. 7 points out a further the actual shapes of the bond-slip resulting curves which, though remaining rather close to the experimental measures, look quite different from the often accepted bi-linear bond-slip relationships.

![Figure 7: Calibrated interface relationships using Chajes et al. (1996) tests (points in figure).](image1)

![Figure 8: Bilinear vs. fracture-based dimensionless shear-slip law. Filled zones characterised by the same areas corresponding to the fracture energy $G_f$ in mode II.](image2)

### 4 COMPARISON WITH THE BILINEAR BOND-SLIP LAW

This section addresses the effect of assuming a bilinear bond-slip relationships instead of the more general interface behaviour resulting by a mechanically-consistent simulation based on fracture mechanics (even though under the simplified hypothesis of pure mode II response).
Thus, a linear softening interface model is defined by means the following expressions

\[
\begin{align*}
\tau[z] &= -k_E s[z] \\
\tau[z] &= -\tau_{y,0} + k_S (s[z] - s_e) \\
\tau[z] &= 0
\end{align*}
\]  

\Rightarrow \begin{align*}
\Rightarrow s[z] \leq s_e \\
\Rightarrow s_e < s[z] \leq s_u \\
\Rightarrow s[z] > s_u
\end{align*}

(9)

where the elastic slip, \( s_e \), is equal to \( \tau_{y,0}/k_E \), while the ultimate one \( s_u = \frac{\tau_{y,0}}{k_E} + \frac{\tau_{y,0}}{k_S} \), being \( k_S \) the slope of the descending branch.

Figure 9: Comparison between the bilinear shear-slip model against the fracture-based proposal: 2 – inches test.

Figure 10: Comparison between the bilinear shear-slip model against the fracture-based proposal: 4 – inches test.

The dimensionless comparison in terms of shear-slip law is shown in Fig. 8. The same fracture energy and the initial elastic stiffness are considered: it follows that the ultimate slip of the bilinear law is equal to \( s_u = \frac{2Gf}{\tau_{y,0}} \).

A complete analytical solution which can be obtained for simulating the behaviour of a FRP strip glued on concrete by assuming the above mentioned bilinear model has been derived in Caggiano et al. (2011).

Figs. 9 to 12 propose the comparison between the numerical simulation, already reported in the previous section, for the four considered tests and the corresponding solutions derived by assuming a bilinear approximation for the bond-slip relationship and the mechanical parameters reported in Table 1.

Finally, the last figures show the superior predicting capacity of the fracture-mechanics based simulation with respect to the bilinear bond-slip relationship, especially in the case of longer anchorage (Figs. 11 and 12). Although the latter leads to rather handy closed-form solutions
for simulating the bond behaviour of FRP strips glued on concrete substrate, this comparative applications points out the limits of such choice for simulating the actual mechanical behaviour of the systems under consideration.

5 CONCLUSIONS

The present paper addressed the issue of modelling the bonding behaviour of FRP strips glued on brittle substrates like those made-out of concrete. Although several “a priori” non-linear bond-slip relationships are available in the scientific literature and often employed for design purposes, this paper provides readers with a more general standpoint for analysing the debonding processes which generally occur at the FRP-to-concrete interface. The proposed model is based on zero-thickness joints for simulating the fracture behaviour of the above mentioned interface. Thus, no bond-slip relationship is assumed “a priori”, but the relevant mechanical parameters describing the elastic and post-elastic behaviour of the interface in the general framework of fracture mechanics are actually considered in the model. The results of the proposed simulation demonstrated the high predictive capacity of the presented model. Moreover, the comparison between those results and the corresponding ones obtained by assuming (as widely-accepted in the scientific literature) a bilinear bond-slip law pointed out the superiority of the proposed approach. Finally, the proposed model leads to more accurate simulations with respect to other approaches characterised by simplified assumptions about the interface behaviour. However, further investigations are needed for better understanding other relevant aspects of the proposed approach, such as the shape of the softening relationship and the as-
sumption of more general fracture patterns. Those two aspects are actually among the future developments of this research.

REFERENCES


