Turbidity current with a roof: Direct numerical simulation of self-stratified turbulent channel flow driven by suspended sediment

Mariano I. Cantero, S. Balachandar, Alessandro Cantelli, Carlos Pirmez, and Gary Parker

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In this work we present direct numerical simulations (DNS) of sediment-laden channel flows. In contrast to previous studies, where the flow has been driven by a constant, uniform pressure gradient, our flows are driven by the excess density imposed by suspended sediment. This configuration provides a simplified model of a turbidity current and is thus called the turbidity current with a roof configuration. Our calculations elucidate with DNS for the first time several fascinating features of sediment-laden flows, which may be summarized as follows. First, the presence of sediment breaks the symmetry of the flow because of a tendency to self-stratify. More specifically, this self-stratification is manifested in terms of a Reynolds-averaged suspended sediment concentration that declines in the upward normal direction and a Reynolds-averaged velocity profile with a maximum that is below the channel centerline. Second, this self-stratification damps the turbulence, particularly near the bottom wall. Two regimes are observed, one in which the flow remains turbulent but the level of turbulence is reduced and another in which the flow relaminarizes in a region near the bottom wall, i.e., bed. Third, the analysis allows the determination of a criterion for the break between these two regimes in terms of an appropriately defined dimensionless settling velocity. The results provide guidance for the improvement of Reynolds-averaged closures for turbulent flow in regard to stratification effects. Although the analysis reported here is not performed at the scale of large oceanic turbidity currents, which have sufficiently large Reynolds numbers to be inaccessible via DNS at this time, the implication of flow relaminarization is of considerable importance. Even a swift oceanic turbidity current which at some point crosses the threshold into the regime of relaminarization may lose the capacity to reentrain sediment that settles on the bed and thus may quickly die as it loses its driving force.


1. Introduction

Turbidity currents in lakes or the deep oceans represent a subset of gravity-driven bottom flows [Allen, 1985; García, 1992]. More familiar gravity-driven bottom flows include those driven by differences in temperature or salinity [Simpson, 1997]. Turbidity currents differ from thermohaline flows, however, in that the agent that renders them heavier than the ambient water around them, i.e., sediment, is nonconservative, and thus can exchange with the bed through erosion and deposition. In this regard, they hold a strong similarity to powder snow avalanches [Hopfinger, 1983].

Direct numerical simulation (DNS) has provided insightful results for lock exchange gravity flows and turbidity currents [see, e.g., Härtel et al., 2000; Necker et al., 2005; Cantero et al., 2007a, 2007b, 2008b]. The ultimate goal of the line of research presented here is the application of DNS to continuous turbidity current dynamics. The complexity of this problem, however, suggests beginning with a somewhat simpler one that retains many of the elements of true turbidity currents. The conceptual model that we use as a basis for the work may be called turbidity current with a roof, hereby shortened to TCR.
2. Problem Setting: Turbidity Current With a Roof

The configuration for the turbidity current with a roof (TCR) concept can be explained as follows. Consider an inclined channel that forms a angle \( \theta \) with the horizontal direction completely submerged in a large, deep tank of stagnant fresh water (see Figure 1). The channel is open to the tank at both ends. In the absence of any density difference between the water in the channel and in the tank there is a hydrostatic pressure distribution acting on the

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**Figure 1.** Problem setting: the flow in the channel is driven by the excess density of the water-sediment mixture. Since the channel is open at both ends to the tank there is no net pressure gradient acting on the flow. The flow represents an idealized case of a turbidity current in which ambient water entrainment is not allowed in the channel.
system and there is no flow in spite of the downward slope of the channel.

[13] Now let a mixture of water and sediment be introduced at the upper end of the channel as shown in Figure 1. This is done in a way that the ambient pressure distribution continues acting on the system so that there is no net pressure gradient imposed along the channel. Under these circumstances the only driving force in the system is the excess density of the water-sediment mixture.

[14] An idealized setting of the problem that represents the section between dashed lines in Figure 1 is a periodic channel (in the streamwise and spanwise directions) with a zero net pressure gradient. The motivation of the setting of the problem is driven by the existence of a statistically steady base flow that is uniform in the streamwise direction, and that has characteristics similar to turbidity currents (TCR model).

[15] In a coordinate system attached to the channel, the equations that govern the flow are [Cantero et al., 2008a]

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho_w} \nabla p + \rho \nabla^2 \mathbf{u} + \frac{\rho - \rho_w}{\rho_w} \mathbf{g},
\]

(1)

\[
\nabla \cdot \mathbf{u} = 0,
\]

(2)

\[
\frac{\partial c}{\partial t} + (\mathbf{u} + \mathbf{V}) \cdot \nabla c = \kappa \nabla^2 c,
\]

(3)

where \( \mathbf{u} = (u, v, w) = (u_x, u_y, u_z) \) is the fluid velocity, \( p \) is the pressure, \( c \) is the volumetric concentration of sediment, \( \mathbf{V} = (V_x, 0, V_z) \) is the settling velocity of the sediment, \( \rho_w \) is the density of the water, \( \rho = \rho_s(1 + Rc) \) is the density of the mixture with \( R = (\rho_s - \rho_w)/\rho_w \) and \( \rho_s \) the density of the sediment, \( \nu \) is the kinematic viscosity of water, \( \kappa \) is the diffusivity of sediment, and \( \mathbf{g} = (g_x, 0, g_z) \) is the acceleration of gravity.

[16] Equations (1)–(3) apply to (1) sediment suspensions that are sufficiently dilute to allow for the use of Boussinesq approximation and the neglect of hindered settling and (2) particles that are sufficiently fine that inertial effects are of second order (see section 3 of Cantero et al. [2008a]).

[17] A “molecular” diffusion term appears in (3). Formally speaking, the term only applies to sediment that is so small that it is subject to Brownian motion. The term is used here in an ad hoc sense as a model for sediment resuspension. Under the right conditions, complex interaction between sediment and near-wall turbulence manifest as an upward flux of sediment particles that are reentrained into the flow. This process is even more complicated when walls are covered with roughness elements of the size of or larger than the sediment particles. The details of these interactions are not well known to date, and the phenomenon of sediment resuspension is treated empirically with entrainment functions based on parameters computed from the mean flow [Garcia and Parker, 1993]. In this work the diffusive flux is set to exactly balance at the walls the depositional flux \( V_zc \) due to settling in order to achieve a statistically steady flow. From the practicalities of numerical calculations, the diffusion term in (3) is also necessary for numerical stability.

[18] The molecular diffusion term is thus a surrogate for the details of particle motion near the wall and the details that arise from the interactions of the particles with the fluid and with each other. In general, the diffusivity should be a function of both particle concentration and local shear [Acrivos, 1995; Foss and Brady, 2000]. In the limit of \( V \rightarrow 0 \) the above governing equations reduce to those corresponding to the transport of a scalar in which case this term accounts for molecular diffusion of the scalar field.

[19] The height of the channel is \( 2h \), the length is \( L_x = 4 \pi h \) and the width is \( L_y = 2 \pi h \). The top and bottom walls of the channel represent a smooth boundary to the flow. The sediment is assumed to be sufficiently fine so that the flow does not allow for net deposition; that is, any particle that settles is instantly reentrained into suspension. Such a condition can be realized, for example, when turbidity currents flow over a bedrock or coarse gravel bed carrying suspended sediment in bypass mode [see, e.g., Piper and Savoye, 1993; Gerber et al., 2008]. The following boundary conditions thus apply:

\[
\mathbf{u} = 0 \text{ at } z = -h \text{ and } z = h,
\]

(4)

\[
c V_z - \kappa \frac{\partial c}{\partial z} = 0 \text{ at } z = -h \text{ and } z = h.
\]

(5)

In the directions tangent to the walls periodic boundary conditions are applied for all variables.

2.1. Mean Flow Equations

[20] It is of considerable value to define the balance of the mean flow averaged over turbulence. Mean variables are obtained by means of a combination of averaging in the plane tangential to the walls (\( x-y \) planes) and in time. The mean variables are denoted by an overbar (\( \bar{\cdot} \) ) and perturbations from the mean are denoted by a prime (\( ' \) ). The problem described above has a fully developed statistically steady state for which the mean flow equations are

\[
\nu \frac{d^2 \bar{\eta}}{d \zeta^2} - \frac{d}{d \zeta} \left( \bar{\omega} \bar{\omega} \right) + R \tau \bar{g}_z = 0,
\]

(6)

\[
\frac{1}{\rho_w} \frac{d}{d \zeta} \left( \bar{\omega}^2 \right) - R \tau \bar{g}_z = 0,
\]

(7)

\[
\kappa \frac{d^2 \tau}{d \zeta^2} - V_z \frac{d}{d \zeta} - \frac{d}{d \zeta} \left( \bar{\omega} \bar{\omega} \right) = 0.
\]

(8)

Observe that by redefining the mean pressure as

\[
p^* = \bar{p} + \rho_w \bar{\omega}^2 - \rho_w R \int_0^\zeta \bar{\tau}(\zeta) d\zeta
\]

(9)

(7) becomes trivial and pressure plays only the role of ensuring incompressibility of the flow field. Note that (6) and (8) are not changed by this definition of mean pressure.
Table 1. Cases Studied in This Work

<table>
<thead>
<tr>
<th>Case</th>
<th>( \hat{V} )</th>
<th>( T_{\text{avg}} )</th>
<th>( u_0 )</th>
<th>( R_{hS} )</th>
<th>( R_{hB} )</th>
<th>( \bar{u}_{avg} )</th>
<th>( \bar{u}_{b} )</th>
<th>( \bar{\tau}_h )</th>
<th>( \bar{\tau}_b )</th>
<th>( \bar{z}_{\text{avg,max}} )</th>
<th>( \bar{z}_{pwc} )</th>
<th>( \bar{z}_{0.05%} )</th>
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</table>

\*For all cases the domain size is \( L_x = 4 \pi h \times L_y = 2 \pi h \times L_z = 2h \) and resolution is \( N_x = 96 \times N_y = 96 \times N_z = 97 \). Here \( \bar{z}_{\text{avg,max}} \), location of streamwise velocity maximum; \( \bar{z}_{pwc} \), location of pycnocline; and \( \bar{z}_{0.05\%} \), location of zero Reynolds shear stress.

Integration of equation (6) in the volume gives

\[
\tau = \rho_u \int \tau \, dz.
\]

since \(\overline{w^2} = 0\) at the walls. Here \( \tau_i \) and \( \tau_b \) are the top and bottom shear stresses. From this equation a velocity scale for the flow can be defined as

\[
u_{avg}^2 = \frac{1}{2} \left( \nu_{avg}^2 + \nu_{b}^2 \right) = \frac{1}{2} \left( \rho_u \frac{\tau_i}{\rho_w} + \frac{\tau_b}{\rho_w} \right).
\]

Observe that

\[
u_{avg}^2 = g_c R \sigma(v),
\]

where

\[
\sigma(v) = \frac{1}{2h} \int_{-h}^{h} \tau \, dz.
\]

is the volume averaged sediment concentration, which is conserved over time.

It can be shown that for the case \( V = 0 \), with the top and bottom boundary conditions employed here (8) yields a uniform \( \tau \). In this case the problem reduces to a pure channel flow with a constant forcing “pressure gradient” equal to \( \rho_w \, R \sigma(v) \, g_c \). Furthermore \( \nu_{avg} = \nu_{avg}^2 - \nu_{avg}^2 \), owing to the symmetry of the problem.

2.2. Dimensionless Equations

Using \( \nu_{avg} \) as the velocity scale, \( h \) as the length scale, \( \sigma(v) \) as the concentration scale and \( T = h/\nu_{avg} \) as the derived timescale, the dimensionless versions of (1)–(3) can be written as

\[
\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} = -\nabla p + \frac{1}{Re_r} \nabla \bar{C}_{\rho} + \bar{C}_{\rho},
\]

\[
\nabla \cdot \bar{u} = 0.
\]

This definition of the pressure, along with the boundary condition \( \partial p/\partial z = 0 \) in the pressure Poisson equation of the time-split scheme, avoids the formation of a pressure boundary layer.

The statistically steady state solution of pure channel flow with uniform particle concentration was used as initial
condition for cases 1, 7 and 10. For the remaining cases the initial condition was the statistically steady state of case 1. Values of $\bar{\sigma}$ and $\bar{u}$ were monitored over the course of the calculations until they reached a statistically steady value. Representative statistics were obtained with an integration time $T_{avg}$ after achieving a statistically steady state. This time of integration is listed in Table 1. The length of the integration time was tested for three cases corresponding to no stratification effects (case 0), intermediate stratification effects (cases 6 and 7), and high stratification effects (case 10). All the cases showed that $T_{avg} = 50$ was appropriate.

[28] The bulk velocity of the flow is defined as

$$u_b = \frac{1}{2h} \int_{-h}^{h} \bar{u}(z) \, dz.$$  

The value of $Re_z$ has been set to 180. This value of $Re_z$ corresponds to a value of $Re_y = u_h h / \nu \approx 3000$. The flow in the channel is thus in the turbulent range. The value of $Sc$ has been set to 1 on the basis of the findings of Necker et al. [2005] and Cantero et al. [2008a], who indicated that the contribution of the diffusion term is of second order as long as it is order 1 or larger. For the cases that remain actively turbulent, which are primarily governed by Reynolds fluxes, the influence of $Sc$ is only relevant in the near-wall regions.

3. Numerical Method

[28] The dimensionless governing set of equations (14)–(16) are solved using a dealiased pseudospectral code [Canuto et al., 1988]. Fourier expansions are employed for the flow variables in the directions tangential to the walls ($x$-$y$), while in the inhomogeneous direction normal to the walls ($z$) a Chebyshev expansion is used. A splitting method is used to solve the momentum equation along with the incompressibility condition [see, e.g., Brown et al., 2001]. A low-storage mixed third-order Runge-Kutta and Crank-Nicolson scheme is used for the temporal discretization of the advection-diffusion terms. This scheme is carried out in three stages with the pressure correction at the end of each stage. More details on the implementation of this numerical scheme can be found in the work by Cortese and Balachandar [1995]. Periodic boundary conditions are enforced for all the variables in the directions tangential to the walls. At the top and bottom walls the conditions of no-slip and zero net flux are enforced for the velocity field and the concentration field, respectively (see (4) and (5)). The domain size and resolution employed for the simulations are $(L_x, L_y, L_z) = (4\pi h, 2\pi h, 2h)$ and $(N_x, N_y, N_z) = (96, 96, 97)$, respectively. Dealiased nonlinear terms are computed in a grid $(3N_x/2, 3N_y/2, N_z)$. Validation of the code for sediment-free flow can be found in the work by Cantero et al. [2006, 2007a, 2007b] and in the context of sediment laden flows in the work by Cantero et al. [2008a, 2008c].

4. Nonstratified Channel Flow

[29] As explained above, the case $V = 0$ (case 0) reduces to pure channel flow, with no internal stratification effects. The solution for this case is well known [e.g., Kim et al., 1987]. These results are nevertheless briefly presented here, because they provide a baseline against which to compare the results in the presence of particle-induced stratification.

[30] Figure 2a presents the mean streamwise velocity profile from the wall to the center of the channel in wall units defined as

$$z^+ = \frac{z u_{a, avg}}{\nu} \quad \text{and} \quad u^+ = \frac{\nu}{u_{a, avg}}.$$  

Note that in case 0 analyzed in this section $u_{a, avg}$ is identical to both $u_{a, z}$ and $u_{a, b}$. The solid line represents the solution for the bottom half of the flow. Also included in Figure 2a is a dotted line describing the top half of the flow, but because the two lines are identical it is not visible therein. This perfect symmetry validates the methodology used for averaging. Also shown in this frame are the law of the wall and the logarithmic law (dashed lines), which are seen to be in correspondence with the DNS results. Figure 2b shows the total shear stress (solid line) $\tau_{tot} = \tau_v + \tau_{Re}$ together with the viscous contribution (dashed line) $\tau_v = \rho_k u_b d n d z$, and the Reynolds contribution (dash-dotted line) $\tau_{Re} = -\rho_k \bar{w} \bar{w}^+$. The total shear stress is linear, as expected for the equilibrium state of the flow. Also included in Figure 2b are the DNS data by Kim et al. [1987] for the Reynolds shear stress. The agreement with our DNS data is quite good. Figure 2c shows the velocity RMS values. Also shown in Figure 2c is the DNS data by Kim et al. [1987], again in good agreement with our DNS data. The main difference is less than 1.7% at the peak value of $u_{rms}$.

[31] Figure 3 shows with lines the streamwise and spanwise one-dimensional energy spectra for a near-wall region ($z^+ \approx 6$) and for a far-wall region ($z^+ \approx 145$) for case 0. In Figure 3 $k_x$ and $k_y$ represent the streamwise and spanwise wave numbers. Figure 3 shows that there is no energy pileup at high wave numbers, and that the energy of the high wave number modes is several orders of magnitude smaller than the energy of the low wave number modes. This shows the adequacy of the resolution used. In the cases where $V \neq 0$, stratification can partially or totally suppress turbulence in the bottom region of the channel, and less resolution is needed. On the other hand, as will be shown later, turbulence can be partially enhanced in the top region of the channel. Figure 3 includes the streamwise and spanwise one-dimensional energy spectra for the same near-top wall region ($z^+ \approx 6$) and far-top wall region ($z^+ \approx 145$) for case 5. Figure 3 shows also that for case 5 there is no energy pileup at high wave numbers and that the resolution used is satisfactory.

5. Regimes of Stratification

[32] The presence of settling particles induces a vertical gradient of the particle concentration, and produces a stable stratification of the flow. In the setting studied in this work, stratification is manifested mainly in two different ways. The first noticeable effect is the generation of a nonuniform driving force skewed toward the bottom wall, which breaks the symmetry of the problem. The other effect of stable stratification is to suppress vertical momentum and mass transport [Turner, 1973]. The magnitude of these effects increases with increasing values of settling velocity and the flows can be separated in two regimes. The first regime,
called turbulent stratified flows, includes cases 1–5, which present some degree of stratification but for which turbulence is still active. The results for flows in this regime are presented in section 6. The second regime, called relaminarized stratified flows, includes cases 6–10 for which relaminarization occurs in the bottom near-wall region. Results for flows in this regime are presented in section 7. The analysis of flow relaminarization and a criterion for the onset of the relaminarized stratified flows regime is deferred until section 8.

6. Turbulent Stratified Flows

6.1. Mean Profiles and Reynolds Fluxes

The effects of stratification on the mean streamwise velocity profiles can be clearly seen in Figure 4a. Two main features, which can be observed in this plot, indicate the effect of increasing stratification. The first one is the loss of symmetry by a gradual deviation of the velocity maxima toward the bottom wall with increasing values of $V$. The change is subtle for cases 1 and 2 corresponding to $V = \gamma/\sqrt{C_4}$ and $V = 10^{-2}$, but more clearly observed for cases 3, 4 and 5 corresponding respectively to $V = 1.75 \times 10^{-2}$, $2 \times 10^{-2}$ and $2.125 \times 10^{-2}$. The exact locations of the velocity maxima are listed in Table 1 as $z_{u, \text{max}}$. The second effect is the deviation from a flat, well-mixed velocity profile observed in case 0 corresponding to $V = 0$ to a more curved, less mixed velocity profile with increasing $V$.

The effects of stratification are also apparent in the mean concentration profiles shown in Figure 4b corresponding to the same cases as Figure 4a. For the cases $V \neq 0$ the concentration increases toward the bottom wall as expected, producing less mixed profiles for increasing $V$. It is observed that the concentration profiles show a somewhat well-mixed
behavior in the top half of the channel where stratification effects are less important, while the profiles show a less mixed behavior on the bottom half. This is most clearly seen for cases 3, 4 and 5. Mixing is induced by wall turbulence, but stratification inhibits turbulence in the bottom half of the channel, which explains the less mixed profiles in this region. Stratification also increases the gradient of concentration at the bottom wall, which is consistent with the imposed boundary conditions.

In Figure 4b the concentration profiles show a change of curvature for cases with $V \neq 0$. A pycnocline (rapid change in the values of $c$) develops for cases 1–5, which separates the flow in two layers. The location of the pycnocline, defined by the maximum value of the concentration gradient, is marked in Figure 4b by open circles and listed in Table 1 as $z_{pyc}$. The result of sharpened density profiles between mixed layers is consistent with previous experimental observations [Moore and Long, 1971; Crapper and Linden, 1974] and numerical results [Armenio and Sarkar, 2002] of similar problems. The pycnocline disappears for larger values of settling velocity as will be shown later.

The two-layer behavior can also be seen in Figure 5, where profiles of $w^c c_0 / V z_b$ are shown. Here $z_b$ represents the bottom concentration and the value of $V z_b$ gives a scale for the sediment settling flux near the bottom wall. This ratio then, gives a relative measure of the turbulent sediment flux, $w^c c_t$, to the sediment settling flux. Larger values of this ratio indicate that the flow has a larger ability to mix the sediment into the flow. The top half of the channel shows lower values of $w^c c_0 / V z_b$, which is counterintuitive since the top half of each of the concentration profiles is better mixed than the bottom half, as seen in Figure 4b. For the top half, a scale for the sediment settling flux is $V z_t$, where $z_t$ is the concentration at the top wall. Figure 6 shows the turbulent sediment flux made dimensionless with the appropriate value for each half. The dimensionless values of $z_b$ and $z_t$ are listed in Table 1. Interestingly, the relative turbulent fluxes are larger in the top half than in the bottom half (note the different vertical scales in Figure 6), which explains the well-mixed profiles at the top half. The values of $w^c c_0 / V z_b$ (shown in the top half and associated to the top scale) increase with $V$, while the values of $w^c c_t / V z_b$ (shown in the bottom half and associated to the bottom scale) show the opposite trend. This is due to the fact that as the top half becomes more depleted of sediment with increasing

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{One-dimensional energy spectra for case 0 (lines) and case 5 (symbols). (a) Streamwise spectrum. (b) Spanwise spectrum.}
\end{figure}
values of $V$ the flow’s mixing ability increases. On the other hand, as stronger gradients start to develop in the bottom half with increasing values of $V$, stratification effects become stronger and the mixing ability of the flow decreases.

Figure 7a shows profiles of the total shear stress, $\tau_{\text{tot}} = -\rho_w \bar{w}' \bar{u}' + \rho_w \nu \bar{d} \bar{u}'/\bar{z}$, for cases 0–5. As expected $\tau_{\text{tot}}$ is linear for case 0. On the other hand, for cases 1–5, $\tau_{\text{tot}}$ becomes convex and the convexity increases with increasing values of $V$. This can be explained by recognizing that the integral of (6) from the top wall to a location $\bar{z}$ gives

$$
\frac{\tau_{\text{tot}}(\bar{z})}{\rho_w \bar{u}'_{\text{avg}}^2} = -\frac{u_{\text{avg}}^2}{\bar{u}_{\text{avg}}^2} + \int_0^{\bar{z}} \tau \, d\zeta.
$$

Thus, $\tau_{\text{tot}}$ is linear only for the case of a constant concentration profile (case 0).

Figure 7b shows profiles of the turbulent momentum flux, $\bar{w}' \bar{u}'$, for cases 0–5 are shown in Figure 7b. Different behaviors are observed in the top and bottom halves of the channel. Maximum values of the magnitude of the turbulent momentum flux decrease with increasing $V$ near the bottom wall, and the opposite occurs near the top wall. The behavior in the bottom half is expected since increasing stratification partially damps turbulence. On the other hand, the trend of increasing peaks of the magnitude of the turbulent momentum flux with increasing stratification in
the top half is somewhat puzzling at first glance. This trend can, however, be explained by evaluating (21) at the locations where \( \bar{u}' \bar{w}' \) peaks in the top half, \( \hat{z}_{thp} \).

\[
\frac{-\bar{u}' \bar{w}'}{u_{*,avg}^2} \bigg|_{z_{thp}} \approx -1 + \int_{z_{thp}}^{1} \tau \, dz.
\]  

(22)

Here the term corresponding to the viscous shear stress evaluated at \( \hat{z}_{thp} \) has been neglected. Also, the ratio \( u_{*,f} / u_{*,avg} \) has been approximated by 1 (exact values are presented in Table 1). It is clearly seen from (22) that smaller values of the integral lead to smaller values of \( -\bar{u}' \bar{w}' \) in the top half of the channel, i.e., increasing peak values of the magnitude of the turbulent momentum flux. Figure 8 shows the concentration profiles in the top half of the channel for cases 0–5. Figure 8 shows that indeed the values of the integral in (22) decrease with increasing stratification.

Figure 9 shows the velocity profiles in wall units, defined as

\[
z_{i/h}^+ \equiv \frac{z u_{*,i/h}}{\nu} \quad \text{and} \quad u_{i/h}^+ \equiv \frac{u}{u_{*,avg}}.
\]  

(23)

since the relevant parameter is the wall shear velocity rather than \( u_{*,avg} \). Figure 9a shows the bottom half and Figure 9b shows the top half of the channel. These plots display three regions that are present for all cases 0–5: an inner linear region, an outer logarithmic region and a core region in the neighborhood of the velocity maximum. The inner linear region does not show any appreciable modification due to stratification in any of the halves. The logarithmic region does not show any appreciable modification either for the top half of the channel. For the bottom half, however, several patterns of variations can be observed in the...
logarithmic region with increasing stratification that are discussed in the next paragraph. Finally, the core region shows some modification for both halves which is mainly related to the fact that the velocity maximum location varies with stratification.

Figure 9a shows that the extent of the logarithmic region diminishes with increasing stratification mainly owing to the lowering of the velocity maximum. The main trend observed in this region is a slope of the velocity profile that increases with increasing stratification in the region of validity of the logarithmic law. The logarithmic law is

\[ u_b^* = \frac{1}{K_b} \ln(z_b^*) + B_b, \]  

where \( K_b \) and \( B_b \) are apparent parameters that are affected by stratification effects. For unstratified flows these parameters take the classical values \( K_b = K = 0.41 \) and \( B_b = B = 5.5 \). Figure 9a shows that the validity of the logarithmic law is maintained for the stratified cases by changing the values of \( K_b \) and \( B_b \). As an example, the best fit corresponding to case 5 is presented in Figure 9a (\( K_b = 0.215 \) and \( B_b = 1.2 \)). The values of these parameters that give the best fit to the velocity profile are listed in Table 1.

The increase in the slope of the velocity profiles in the region of validity of the logarithmic law has been observed experimentally in the presence of suspended sediment by Vanoni [1946] and Einstein and Chien [1955] [see also Winterwerp, 2001].

As observed in Figure 4a stable stratification increases the mean streamwise velocity. A measure of the bulk change is the friction coefficient defined as

\[ C_f = 2 \frac{u_{*,\text{avg}}}{u_b^*}. \]  

Figure 8. Concentration profiles for cases 0–5 in the top half of the channel.

Figure 9. Mean velocity profiles in wall units for cases 0–5. (a) Bottom half of the channel. (b) Top half of the channel.
corresponding to case 0 to $C_f = 6.9 \times 10^{-3}$ corresponding to case 4. The corresponding value of the bulk Richardson number, defined as

$$Ri_b = -\frac{g}{u_b^2} \frac{\bar{\nabla} \cdot \bar{\nabla} \bar{h}}{\nabla \cdot \bar{h}},$$  \hspace{1cm} (26)$$

increases from $Ri_b = 0$ corresponding to case 0 to $Ri_b = 0.067$ corresponding to case 4. This behavior has been observed in previous works on forced stratification by active scalars in turbulent channel flow [Armenio and Sarkar, 2002; Taylor et al., 2005]. For example, Armenio and Sarkar [2002] report a decrease of $C_f$ from $C_f = 8.18 \times$
10^{-3} corresponding to $Ri_h = 0$ to $C_f = 6.73 \times 10^{-3}$ corresponding $Ri_h = 0.064$.

6.2. Turbulent Intensities and Anisotropy

Figure 10 shows profiles of $u_{rms} = \sqrt{\langle u'^2 \rangle}$, $v_{rms} = \sqrt{\langle v'^2 \rangle}$ and $w_{rms} = \sqrt{\langle w'^2 \rangle}$ for cases 0–5. The streamwise component plotted in Figure 10a shows little change with stratification for cases 1, 2 and 3. The main change is the displacement of the minimum from the center of the channel toward the bottom wall, which is consistent with the displacement of the streamwise velocity maximum toward the bottom wall with increasing stratification. Cases 4 and 5 show a different behavior at the peak locations. The values of the $u_{rms}$ peaks decrease in the bottom half, which is expected as stronger stratification acts to damp the turbulence. On the other hand, the peak value in the top half increases. This fact is related to the increase of turbulent momentum flux magnitude in this region documented in Figure 7b and explained in section 6.1. For case 5 the peak location in the bottom half is shifted upward showing a stronger influence of stratification. The effect of stratification is more apparent in the spanwise and wall normal components of the velocity RMS shown in Figures 10b and 10c, respectively. These components are reduced by stratification in the bottom half and increased in the top half.

Figure 11 shows a plot of $w_{rms}/u_{rms}$. This parameter provides one measure of the anisotropy of the flow, in so far as the ratio should be unity for perfect isotropy. In the upper half of the flow, stratification has little effect on the degree of anisotropy. In the middle and lower parts of the flow, the effect of stratification is to increase the degree of anisotropy. This is intensified in the logarithmic zone of the bottom half.

7. Relaminarized Stratified Flows

In this section we focus on results for cases 6, 7, 8, 9 and 10 corresponding to $\epsilon = 2.3 \times 10^{-2}, 2.5 \times 10^{-2}, 3 \times 10^{-2}, 3.5 \times 10^{-2}$, and $5 \times 10^{-2}$, respectively. For these cases the flow is more strongly stratified in the vertical than for cases 0–5, with relatively large values of sediment concentration near the bottom wall (see dimensionless values of $\bar{c}_b$ in Table 1). Although the large bottom concentration increases the driving force of the flow near
the bottom wall, the large concentration gradients that also form there damp the turbulent intensities. As a consequence turbulence is strongly suppressed and, as will be explained below, flow relaminarization occurs in the bottom region of the channel. Thus in these cases the velocity and concentration profiles in the bottom half of the channel tend to the laminar solution.

7.1. Laminar Solution

[46] In the context of finite size particles the laminar solution of equations (1)–(3) (or equivalently (14)–(16)) is unrealistic, since particles that are too large to be subject to Brownian motion cannot be sustained in suspension in the absence of turbulence. This set of equations, however, has a solution for laminar flow which is of interest, for example, in the case of electrically charged, small Brownian particles that feel a settling velocity in the presence of an electric field.

[47] Consider a one-dimensional, steady state, uniform flow, i.e., $\tilde{u} = 0$, $\partial / \partial \tilde{t} = 0$, $\partial / \partial \tilde{x} = 0$ and $\partial / \partial \tilde{y} = 0$. Under these circumstances it can be easily shown from (15) that $\tilde{w} = 0$, and that the set of equations (14)–(16) reduces to obtain

$$\frac{d^2 \tilde{u}}{d \tilde{z}^2} = -Re \tilde{c}$$

(27)

and

$$\tilde{V}_z \frac{d \tilde{c}}{d \tilde{z}} = \frac{1}{Re Sc} \frac{d^2 \tilde{c}}{d \tilde{z}^2}.$$  

(28)

Employing the boundary conditions (5), (28) can be integrated to

$$\tilde{c} = A \exp(\tilde{V}_z Re Sc \tilde{z}).$$

(29)

Then,

$$\tilde{c} = \frac{\tilde{V}_z Re Sc}{\sinh(\tilde{V}_z Re Sc)} \exp(\tilde{V}_z Re Sc \tilde{z}),$$

(33)

and

$$\tilde{u} = -\frac{1}{\tilde{V}_z Sc \sinh(\tilde{V}_z Re Sc)} \left[ \exp(\tilde{V}_z Re Sc \tilde{z}) - \tilde{z} \sinh(\tilde{V}_z Re Sc) - \cosh(\tilde{V}_z Re Sc) \right].$$

(34)

[48] Observe that for the case $\tilde{V}_z \to 0$ the relations reduce to expected forms:

$$\tilde{c} = 1 \quad \text{and} \quad \tilde{u} = -\frac{Re_c}{2} (\tilde{z}^2 - 1).$$

(35)

For the case $\tilde{V}_z \to -\infty$

$$\tilde{c} = \delta(\tilde{z} + 1), \quad \text{and} \quad \tilde{u} = 0,$$

(36)

where $\delta$ is the Dirac delta function. The velocity profile is rigorously zero for $\tilde{V}_z \to -\infty$ because of the no-slip boundary condition at the bottom wall. However, for any finite, large and negative values of $\tilde{V}_z$, the velocity solution resembles Couette flow with a quasi-linear profile and with its maximum, $\tilde{u}_{\text{max}}$, at a location, $\tilde{z}_{\text{max}}$ that is very close to the bottom wall. For $\tilde{V}_z \to -\infty \tilde{u}_{\text{max}} \to 0$ and $\tilde{z}_{\text{max}} \to -1$. This behavior of the laminar solution is shown clearly in Figure 13.

7.2. Marginally Turbulent Solution

[50] Figure 14a shows the streamwise velocity profiles for cases 6–10. With increasing values of $V$, the velocity profiles deviate from the sharper turbulent profile of case 0 (see Figure 4) and present a maximum that moves close to the bottom wall. This trend is qualitatively similar to cases 1–5. However, for cases 6–10 the streamwise velocity profile also shows a change of curvature above the maximum which was not observed for cases 1–5. Figure 14a also includes the laminar solution corresponding to $V = 5 \times 10^{-2}$ shown as a dotted line. It is clearly observed therein that the DNS solution for case 10 tends to the laminar solution in the bottom, near-wall region of the channel. It can also be observed from the slope of the velocity profiles that the shear at the bottom wall increases with increasing $V$. This trend is consistent with the progressive focusing of the particle concentration and the driving force of the flow toward the bottom wall.

[51] In the top part of the channel ($-0.5 < z$) the velocity profiles show a well-mixed behavior. Stratification effects due to concentration gradients are negligible in this region of the channel and wall turbulence is active. The main effect noticed is the reduction of the top wall shear stress depicted by the decrease in the velocity slopes near the top wall. This is expected, however, in order to compensate for the increase of shear at the bottom wall so as to give $\tilde{u}_{b, \text{avg}} = 1$ (see (11)).

[52] Interestingly, cases 6–10 show a decreasing trend of the streamwise velocity and consequently a reduction of $\tilde{u}_b$.
(and an increase of $C_i$) with increasing values of $V$. This is opposed to the trend observed for cases 1–4 (see Table 1). It can be easily shown that $\tilde{u}_b$ computed from (34) for the laminar solution satisfies $\tilde{u}_b \to 0$ as $\tilde{V}_z \to -\infty$. The reduction of $\tilde{u}_b$ in the marginally turbulent solutions with

**Figure 13.** Laminar solution for several values of the settling velocity. (a) Velocity profiles. (b) Concentration profiles. The inset frame in Figure 13b shows the near-wall details of the solution.

**Figure 14.** Mean profiles for cases 6–10. The laminar solutions are shown as dotted lines. (a) Mean streamwise velocity profiles. Laminar solution for case 10 is included. (b) Mean concentration profile. Laminar solutions for cases 6 and 10 are included. See case references in Figure 14a.
increasing values of $V$ is consistent with this fact. Integration of the streamwise component of the buoyancy term in (14) shows that the net driving force of the flow is the same for all cases 0–10. However, with increasing values of $V$, the sediment increasingly accumulates near the bottom wall inducing a large driving force over a thinner region of the flow close to the bottom wall. This redistribution of the driving force causes a reduction in the flow discharge for cases 7–10.

[53] Figure 14b shows the concentration profiles for the same cases as Figure 14a together with the laminar solutions for cases 6 and 10 shown as dotted lines. It is clearly observed in Figure 14b that there is a very good agreement between the marginally turbulent solutions and the corresponding laminar solution close to the bottom wall. For case 10 the agreement is very good for the whole depth, with the largest difference smaller than 3% in the central region of the channel. In this case the particle concentration is practically zero above $-0.5 < z < 1$, and the driving force of the flow is considerably decreased in this region. As explained before, the increased concentration near the bottom wall increases the driving force of the flow near the bottom wall. The result is a velocity profile observed in Figure 14a that has a prominent peak near the bottom wall, which in turn induces a very sharp velocity gradient in this region.

[54] Figure 15 shows profiles of the velocity RMS values in the three directions for cases 6–10. With increasing stratification the RMS values decrease strongly near the bottom wall, and the local maximum in the bottom half of the channel disappears for cases 8, 9 and 10. Interestingly, the local maxima in the top half of the channel for cases 6–10 show the opposite trend to cases 0–5. For settling velocities larger than $V = 2.3 \times 10^{-2}$ corresponding to case 6, the peak value decreases with increasing values of $V$. This decrease in the top half peaks of the RMS values is associated to the reduction of $u_{r, v}^2 / u_{r, avg}^2$with increasing $V$ as presented in Table 1.

8. Flow Relaminarization Below the Velocity Maximum

[55] Figures 10 and 15 show a tendency for the turbulence near the bottom wall to be suppressed as the value of $V$ is increased. For example, in case 10 turbulence intensity in the bottom half of the channel, measured in terms of peak values of $u_{rms}$, $v_{rms}$ and $w_{rms}$, is only 16%, 27% and 12% of the corresponding values in the unstratified case (case 0), respectively.

[56] The ratios of the turbulent momentum stress ($\tau_{Re} = -\rho_{2}u'\v' / \rho_{2}$) to the viscous stress ($\tau_{r} = \rho_{2}u' (d\gamma / dz)$), and the turbulent sediment flux ($F_{Re} = -c'w'$) to the viscous sediment flux ($F_{r} = \kappa d\gamma / dz$) are shown in Figures 16a and 16b, respectively, for cases 0, 5, 6 and 10. In Figure 16a the striking observation is that in all cases the ratio $\tau_{Re} / \tau_{r}$ separates into an upper layer and a lower layer with the ratio rapidly diverging at the interface between the two layers. This behavior is due to the denominator $\tau_{r}$ becoming zero at the location of mean streamwise velocity maximum. The only exception is case 0 where both the mean streamwise velocity maximum and zero turbulent momentum stress occur at the channel center. As a result, since both the numerator and the denominator simultaneously approach zero, the ratio $\tau_{Re} / \tau_{r}$ remains bounded across the entire channel.

[57] With the addition of stratification, the symmetry is broken and the location of mean streamwise velocity maximum does not coincide with the location of zero turbulent momentum stress, resulting in a singular value for the ratio $\tau_{Re} / \tau_{r}$ where $d\gamma / dz = 0$. Since the location of maximum streamwise velocity shifts toward the bottom wall, with increasing $V$ the upper layer expands at the expense of the bottom layer. For example, in case 10 the bottom layer covers only one eighth of the channel, between $-1 < z < 0.75$. The zero crossing of the ratio (best seen in case 5) occurs in the bottom layer, suggesting that the point of zero turbulent momentum stress ($\tau_{Re} = 0$) occurs below the location of mean streamwise velocity maximum (see Table 1). In cases 6 and 10, the ratio $\tau_{Re} / \tau_{r}$ is practically zero over the entire lower layer. A corresponding two-layer behavior can be observed in Figure 16b for the ratio $F_{Re} / F_{r}$. In particular, this ratio becomes virtually zero in cases 6 and 10 within the lower layer. The ratios $\tau_{Re} / \tau_{r}$ and $F_{Re} / F_{r}$ measure the relative importance of turbulent momentum and mass fluxes with respect to the viscous counterparts. A negligible ratio indicates that viscous fluxes overwhelm the turbulent counterparts, and thus that the flow has relaminarized in that region. Such a relaminarized region exists for all cases 6–10 below the location of mean streamwise velocity maximum. Note that, as can be observed from Figures 10 and 15, turbulent intensities are not completely suppressed below the mean streamwise velocity maximum. Here the concept of relaminarization is used in a loose sense meaning that the vertical Reynolds fluxes are practically suppressed to zero as can be observed in Figures 16a and 16b. Thus, while velocity fluctuations (predominantly streamwise in direction) persist in the lower layer, vertical flux of momentum and concentration due to velocity fluctuations is completely shut down. Such relaminarization of wall turbulence has been observed in the context of polymer additives [Kim et al., 2007, 2008].

[58] The reason for relaminarization below the velocity maximum in cases 6–10 is suppression of the turbulence due to the effect of density stratification. The upward normal variation of the mean concentration of suspended sediment ($\bar{c}$) is shown in Figure 14b. It can be inferred from Figure 14b that the gradient of mean sediment concentration increases strongly with $V$ in the region near the bottom wall and has a strong effect of wall turbulence.

8.1. Criterion for Onset of Relaminarization Below the Velocity Maximum

[59] Turbulence damping associated with density stratification is typically characterized in terms of the gradient Richardson number or the flux Richardson number [Turner, 1973] defined as

$$Ri_{g} = \frac{g_{e} \delta c / \bar{c}}{(d\gamma / dz)^{2}}$$

$$Ri_{f} = \frac{R(g_{e} c'w' + \bar{c} c'w' / u' w')}{{u'}^{2} (d\gamma / dz)}$$

respectively. Figure 17 shows $Ri_{g}$ for cases 1, 5, 6 and 10. Figure 17a shows the top region from the top wall down to the velocity maximum and Figure 17b shows the bottom part from the velocity maximum down to the bottom wall. Figures 17a and 17b show also a dotted vertical line.
indicating the critical value for linear stability ($0.25 < Ri_g$). The parameter $Ri_g$ diverges at the velocity maximum, and in regions close to the velocity maximum it reaches large values. As expected, near the bottom wall the values of $Ri_g$ increase with increased stratification effects, i.e., with increasing values of $V$. In cases 1 and 5, $Ri_g$ grows slowly with increasing $z/h$ until it reaches a value of 0.1. Beyond this point $Ri_g$ increases very rapidly with increasing $z/h$. On the other hand, for cases 6 and 10 $Ri_g$ increases very rapidly with $z/h$ for the entire range plotted. Near the top wall $Ri_g$ does not increase as fast with increasing values of $V$, and is maintained below the critical value 0.25 for all cases in regions away from the velocity maximum. The behavior of $Ri_g$ is similar and is thus not shown.

Figure 15. Turbulence statistics for cases 6–10. (a) Streamwise component. (b) Spanwise component. (c) Vertical component.
Unfortunately, the fact that $Rig$ grows very rapidly and beyond the critical value in the neighborhood of the velocity maximum diminishes the ability of $Rig$ to clearly indicate regions of the flow where stratification is important. Figure 18 shows profiles of the Brunt-Väisälä frequency $N = \left(\frac{gRd\sigma}{dz}\right)^{0.5}$ for cases 1, 3, 5, 6 and 10. Two different behaviors can be clearly identified in the top and bottom Figure 17. Gradient Richardson number. (a) The region from the top wall down to the velocity maximum and (b) the region from the velocity maximum down to the bottom wall. The dotted vertical lines indicate the critical value for linear stability ($0.25 < Rig$).

Figure 16. (a) Profile of $\tau_{Re}/\tau_v$. (b) Profile of $F_{Re}/F_v$. 
halves of the channel. In the top half $N$ decreases from the wall down and it presents values $N/(u_{*,avg}/h) < 3$ for all cases presented in Figure 18. In the bottom half $N$ presents larger values than in the top half for cases 3–10, and for cases 6 and 10 $N$ increases very rapidly from $z/h < -0.3$ dawn to the bottom wall. The inset in Figure 18 presents the details for the near-wall region.

Another approach to quantifying the effect of stratification is to compare the Ozmidov length scale, $l_o = (\epsilon/N^3)^{0.5}$, to the Kolmogorov length scale, $l_K = (\nu^3/\epsilon)^{0.25}$. Here $\epsilon$ is the dissipation rate of turbulent kinetic energy. The Ozmidov scale is an estimate of the smallest scale influenced by buoyancy, and the Kolmogorov scale is an estimate for the smallest turbulent scale. Figures 19a and 19b show $l_K/h$ and $l_o/h$ for cases 1, 4, 5, 6 and 10, respectively. The profiles for cases 2–3 and cases 7–9 are not shown in Figures 19a and 19b to allow for better readability. Profiles corresponding to cases 2–3 lay in between the profiles for cases 1 and 4, and the profiles corresponding to cases 7–9 lay in between the profiles corresponding to cases 6 and 10. The Kolmogorov scale in Figure 19a presents two very different behaviors below the velocity maximum, one corresponding to the cases that remain turbulent (cases 1–5) and the other corresponding to the cases that relaminarize below the velocity maximum (cases 6–10). While for cases 1–5 the Kolmogorov scale decreases downward from the velocity maximum, for cases 6–10 it increases, reaching values above $0.02h$. Above the velocity maximum the behavior of the Kolmogorov scale is similar for all cases. The Ozmidov scale in Figure 19b also presents different behaviors for cases 1–5 and cases 6–10. Above the velocity maximum, cases 6–10 present larger values than cases 2–5, which implies that in the former case stratification effects are relegated to larger scales in this region of the flow. Below the velocity maximum cases 6–10 present values of $l_o$ smaller than $0.01h$ (see inset in Figure 19b), while cases

Figure 18. Profiles of $N$. The inset frame shows the near-wall details.

Figure 19. (a) Profile of $l_K/h$. (b) Profiles of $l_o/h$. The inset in Figure 19b shows the details in the region below the velocity maximum for the cases plotted.
1–5 present values that are larger than 0.025h except very close to the bottom wall, where $\epsilon$ is very large.

[62] The evidence presented in Figures 19a and 19b shows that $l_0 < l_k$ below the velocity maximum for cases 6–10. Thus, all the length scales present in the flow are strongly affected by stratification effects below the velocity maximum for these cases. On the other hand, for cases 1–5, $l_0 > l_k$ below the velocity maximum, and stratification effects are not felt on all the scales present in the flow below the velocity maximum. Here it must be cautioned that $l_k$ and $l_0$ provide only order of magnitude estimates. Nevertheless, the behavior of $l_k$ and $l_0$ presented in Figures 19a and 19b show a clear change from case 5 to case 6. On the basis of this analysis, it can be concluded that relaminarization below the velocity maximum occurs for a critical value of $|V_z|/u_{b,h}$ within the interval $2.22 \times 10^{-2} < |V_z|/u_{b,h} < 2.43 \times 10^{-2}$, which are the values of $|V_z|/u_{b,h}$ corresponding to cases 5 and 6, respectively.

8.2. Implications for Turbidity Currents at Field Scale

[63] The fact that cases 6–10 present strongly damped vertical Reynolds fluxes (see Figure 16) means that turbidity currents composed solely of such coarse sediment are likely to be unsustainable in nature. In the context of the present formalism, particles are kept in suspension even when the turbidity current relaminarizes below the streamwise velocity maximum in order to sustain the flow. This is done in an artificial manner by means of the “molecular” diffusivity of the particles. In natural turbidity currents on the other hand, the inability of the flow near the bed to maintain itself in a turbulent state would cause the sediment to rain out on the bed. This would cause the current to die, as it would lose its driving force.

[64] Field scale turbidity currents, however, are composed of a wide range of sediment sizes ranging from clay to coarse sand. On the basis of the present results, it can be speculated that a turbidity current composed of “fine” and “coarse” sediment may actually be able to flow depending on the relative fraction of fine to coarse sediment. If the fine fraction is large enough, the turbidity current may be able to transport the coarse sediment for very long distances without sustained settling of the fine material. In such a situation the fine fraction provides the driving force to the flow, and maintains large vertical turbulent fluxes that keep the coarse sediment in suspension as well. In a situation for which the fine fraction is small, relaminarization of the flow may result in a massive deposit of the coarse fraction.

9. Conclusions

[65] This work describes the application of direct numerical simulation to the solution of a novel problem, i.e., the turbidity current with a roof. This configuration consists of a mildly tilted channel flow carrying a dilute suspension of particles with a finite fall velocity. The flow is maintained solely by gravity pulling the particles, and thus the flow downslope. Such a flow is not only self-stratifying, but is also self-driving. As such, it captures much of the essence of turbidity currents.

[66] No-flux boundary conditions for sediment at the bottom and top walls combined with the finite fall velocity ensure the development of a mean streamwise velocity profiles with a maximum that is pushed below the centerline of the channel. It also ensures maximum suspended sediment concentration at the bottom boundary that declines away from the wall in the upward normal direction, a condition that ensures stable stratification of the flow.

[67] Of particular interest here is the effect of self-stratification on the flow. This self-stratification is found to increase as an appropriately defined dimensionless particle fall velocity increases. This dimensionless parameter is defined as the ratio $|V_z|/u_{b,h}$ of wall normal component of the fall velocity to the shear velocity acting on the bottom wall. When $|V_z|/u_{b,h}$ is sufficiently small, the effect of increasing stratification is simply to suppress turbulence as measured by quantities such as turbulent shear stress and turbulent concentration flux of suspended sediment. This suppression is realized most strongly in the lower part of the flow below the mean streamwise velocity maximum.

[68] Beyond a specific threshold value, further increases in $|V_z|/u_{b,h}$ result in a near-complete relaminarization of the flow in the lower layer below the point of mean streamwise velocity maximum. Our results show that for a specific simulation of $Re_z = 180$ at a bed slope of $\theta = 5^\circ$ the threshold value lies in the interval $2.22 \times 10^{-2} < |V_z|/u_{b,h} < 2.43 \times 10^{-2}$. This relaminarization is reflected in near vanishing of both the turbulent shear stress and turbulent flux of suspended sediment in the flow region below the mean streamwise velocity maximum. The flow in the upper layer remains turbulent, but carries very little suspended sediment. That is, the upper turbulent flow is simply dragged downstream by a sediment-laden flow in the lower layer that is so strongly self-stratified as to be essentially nonturbulent. The critical value of $|V_z|/u_{b,h}$ at which the transition occurs is likely to depend on the Reynolds number of the flow, and in particular it can be expected that with increasing intensity of turbulence (increasing $Re_z$) the threshold particle settling velocity for relaminarization can be expected to increase. Preliminary calculations at $Re_z = 400$ show that strong turbulence damping still occurs with increasing $V_z$, but relaminarization is delayed to larger particle settling velocities.

[69] The results presented in this paper have important implications for Reynolds-averaged models of turbulent flow [Mellor and Yamada, 1982; Rodi, 1984] that describe the effect of stratification through empirical closures. They have even more important implications for field turbidity currents, which may have velocities in the order of meters per second and layer thicknesses of tens of meters. The results presented here are admittedly at much lower Reynolds numbers than those corresponding to field conditions. The limitation on Reynolds number comes from our desire to perform direct numerical simulation without involving ad hoc closure assumptions. Nevertheless, one can appeal to approximate Reynolds invariance in the fully turbulent regime to suggest that the results presented here are of relevance and can be extrapolated in a loose way to field conditions.

[70] Under certain conditions, such as those posed by, e.g., a downstream decline in slope, it appears that a region close to the bottom boundary of a turbidity current may self-stratify to the point that vertical Reynolds fluxes are killed. Upon reaching this condition, the turbidity current would lose its ability to reentrain sediment as it settles on the bed.
As a result, the current would eventually die as it loses its driving force producing a massive deposit.

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